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In-plane shear loading of granular membranes modeled as a Lagrangian assembly of rotating elastic particles

Emilio Turco*

Department of Architecture, Design and Urban planning (DADU), University of Sassari, Italy

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ABSTRACT

A micro-mechanical model devoted to study large deformations of cohesive granular media subjected to quasi-static external actions is presented and discussed. The model, leaving out dynamical effects, is completely described by the definition of the strain energy of the interaction between two nearby grains. The solution of the nonlinear equilibrium system of equations is searched in the framework of a stepwise analysis which uses Newton's iterative strategy as the predictor and Rik's arc-length idea as the corrector. Selected numerical simulation results are discussed to highlight the main peculiarities of the model and the analysis strategy of the obtained results. The discussions suggest the necessary next steps for tackling more complex problems such as those which consider the failure of grain interactions.

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1. Introduction

Technical applications of mechanical models that predict the behavior of granular media span over many fields. Besides being relevant in geotechnics, *e.g.* for analyzing certain kinds of earth structures, powder processing [1] and pharmaceutical tablet forming [2], the mechanics of granular media is also useful for modeling granular membranes whose applications span from civil engineering to biomembranes [3]. In 1979 Cundall and Strack proposed to model the mechanical behavior of grain packages as assemblies of mutually interacting discs, see [4]. This idea, which has proven to be still relevant, is also the basis of the model discussed in [5] as well as in recent publications for cohesionless granular materials such as [6,7]. Experiments, although rare, that show that the mechanical behavior of a grains package could potentially be predicted by using such discrete models range from [8] to [9]. Remarkably, in the last few years, several papers have been published concerning 3D beams, see [10], 2D networks of beams, see [11], and plates, see [12], which have many similarities with the models used for describing grains packages. The keynote is the description of such structural elements as an assembly of rigid links and elastic joints. This idea dates back to 1929 when Hencky, see [13], estimated the buckling load of two-dimensional beams using such an approach. Almost one hundred before Hencky, Piola, see [14], had already suggested the use of a discrete model for Euler's beam. The aim of the present work is to develop a novel Lagrangian model able to describe the quasi-static deformation behavior of packages of grains. To this end, following the route traced in the foregoing efforts, it is assumed that each grain behaves as rigid whereas the elastic parts, the joints, are localized in their interfaces. Leaving out dynamical effects, the mechanical interaction between two nearby grains is completely described in a variational framework by assuming a suitable form for the strain energy of the interaction, see Section 2. Response under quasi-static conditions is of wide practical interest with respect to the interpretation of many experimental techniques used to characterize materials with granular structures. As reported in [5], in the widely used discrete element method, these equations are solved by means of explicit time-integration schemes and utilize fictitious damping parameters and other scaling rules. Such strategies to recover quasi-static solutions from dynamic methods can be erroneous for nonlinear systems where the uniqueness of the solution is not guaranteed. Consequently, the reliability of computed mechanical behavior of the grains packages can be questionable from both quantitative and qualitative points of view. Here, taking into account that quasi-static simulations with discrete models generally pose numerical challenges associated with certain instabilities and bifurcation points, a stepwise procedure using Newton's method as predictor and Riks' arc-length idea as corrector is applied to reconstruct the equilibrium path getting around the possible critical points, see Section 3. The discussion of the results relative to the presented numerical simulations permits to highlight the strengths of the model and of the used strategy of analysis, see Section 4, besides suggesting future challenges, see Section 5.

* Corresponding author.
E-mail address: emilio.turco@uniss.it

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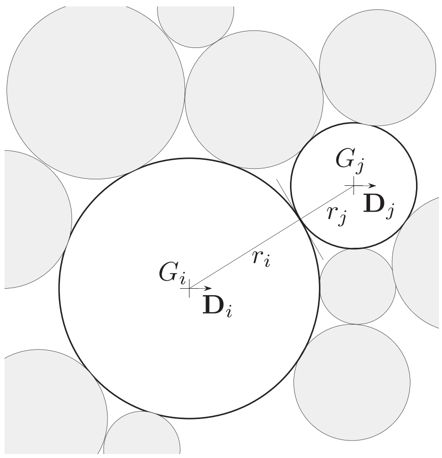

Fig. 1. Geometry of two nearby grains in a granular media.

2. Elastic interactions between two particles

As we will show in details in Section 3, we tackle the system of nonlinear equilibrium equations by using a stepwise procedure. Leaving out dynamical effects, the basic tool required in the aforementioned approach is the definition of the strain energy of the system of grains, that is, using the additivity property of the energy, the strain energy of the single interaction.

We consider two nearby grains such as those sketched in Fig. 1. For the sake of simplicity, the grains i and j are considered of circular shape, their centers (in the reference configuration) and radii are G_i, r_i and G_j, r_j, respectively. Furthermore, in order to describe the rotation of each one of the two nearby grains, two unit vectors $\mathbf{D}_i$ and $\mathbf{D}_j$ are also reported (in this case, we choose $\mathbf{D}_i = \mathbf{D}_j$ aligned with the horizontal direction, see Fig. 1, but any other choice is equivalent).

We can describe the relative motion of each grain-pair by considering as kinematic parameters, the current positions of the centers g_i and g_j, and the rotations of each one grain around their centers, α_i and α_j, or considering the unit vectors $\mathbf{d}_i$ and $\mathbf{d}_j$ which are the correspondent to $\mathbf{D}_i$ and $\mathbf{D}_j$, respectively. Fig. 2 reports the reference and the current configurations of two grains. Following the route traced by Piola and Hencky, recently used to model microbeams, see [11], we can distinguish, see again Fig. 2, three strain measures: *(i)* the stretching $\Delta\ell$, *(ii)* the bending $\Delta\beta$ and *iii)* the sliding $\Delta\gamma$.[1]

The strain measures $\Delta\ell$, $\Delta\beta$ and $\Delta\gamma$, can be written as:

$$\Delta\ell = \|g_j - g_i\| - r_i \cos\beta_i - r_j \cos\beta_j \,, \tag{1}$$

$$\Delta\beta = \beta_j - \beta_i \,, \tag{2}$$

$$\Delta\gamma = r_i \sin\beta_i + r_j \sin\beta_j \,, \tag{3}$$

taking into account that the angle β_k $(k = i, j)$ can be expressed as $\beta_k = \alpha_k + \alpha_0 - \bar{\alpha}$, see again Fig. 2, while their sinuses and cosines can efficiently be computed by using trigonometrical addition formulae. Elastic interactions can completely be described by the strain energy elementary contribution E_e associated to the aforementioned strain measures relative to a grain-pair. The simplest form of the strain energy that maybe considered is the quadratic diagonal form:

$$E_e = \frac{1}{2}\left(a\Delta\ell^2 + b\Delta\beta^2 + c\Delta\gamma^2\right), \tag{4}$$

where a, b and c are three constitutive parameters which values should be determined by means of some suitably chosen experimental tests. The literature is rather dense with the form of strain energy expressions that maybe utilized for grain-pair interactions including for modeling failure, see for example [15].

Following remarks are in-order with respect to the strain energy defined in the foregoing: *(i)* clearly the form (4) is not the only possible form since we can define the bending part in a different way, see, *e.g.*, [16], or the stretching and the sliding using the suggestions reported in [17]; *(ii)* Eqs. (1)–(3), do not introduce any kinematic approximation that are often introduced in many other formulations of discrete models; and *(iii)* the Lagrangian model immediately above described depends on the constitutive parameters a, b and c whose values, in general, vary from grain-pair to grain-pair.

[1] Using a geotechnicians' language, the bending measure is often called rocking effect.

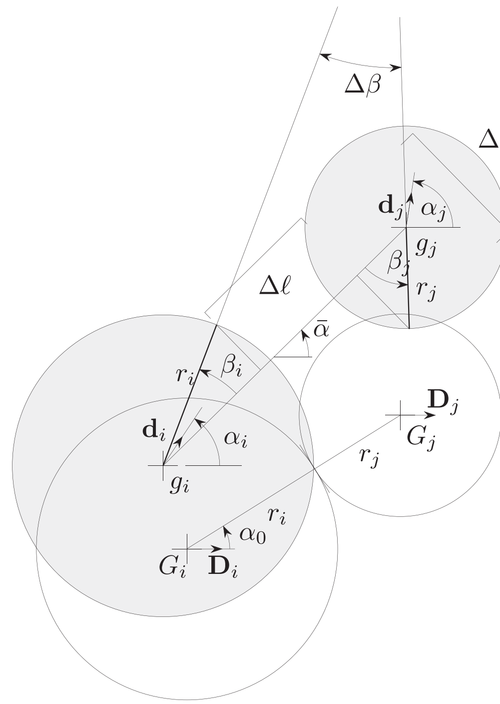

Fig. 2. Reference (white) and current (grey) configuration of two grains: geometric parameters and strain measures.

3. Recovering the equilibrium path

The elastic interaction described by the strain energy (4) can be used to build an algorithm based upon a stepwise procedure. If we consider, as we will do in this work, that there are not external forces but only given displacements on grains then the nonlinear system of equilibrium equations reads:

$$\frac{dE(\mathbf{u}, \lambda\bar{\mathbf{u}})}{d\mathbf{u}} = \mathbf{s}(\mathbf{u}, \lambda\bar{\mathbf{u}}) = \mathbf{0}\,, \tag{5}$$

being E the total strain energy obtained by summing its elementary contributions, defined by Eq. (4), and $\mathbf{s}$ is the reaction depending on $\mathbf{u}$ and $\lambda\bar{\mathbf{u}}$ that is from two vectors which collect the free and the assigned displacements, ruled by the non-dimensional parameter λ, respectively.

We remark that the elementary contribution to the reaction $\mathbf{s}$ can be computed in an exact way by means of differentiation.

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