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An efficient transient analysis method for time-varying structures based on statistical energy analysis

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ABSTRACT

The statistical energy analysis (SEA) is applied to predict the transient energy response of structures with time-varying parameters for the first time. With the energy governing equations derived by considering time-varying SEA parameters and energy flow item caused by time-varying damping loss factor, SEA method is firstly applied to predict transient energy response of time-varying systems. Then, numerical examples of a time-varying two-oscillator system, a time-varying L-shaped fold plate and a complex time-varying vibro-acoustic structure are investigated to demonstrate the effectiveness and accuracy of SEA method for time-varying system. The Newmark-beta method and finite element method are used to verify the accuracy of predicted transient energy response. Results show that SEA method for time-varying system is capable of predicting transient energy response of time-varying structures with sufficient accuracy and also applicable for transient analysis of complex time-varying structures with a small computational cost.

1. Introduction

Many structures in practical engineering are time-varying structures, which are characterized by the mass, stiffness or damping properties that change with time. For instance, the rapid combustion of rocket fuel will lead to time-varying mass dynamic problems [1] and the aerodynamic heating will change the stiffness and damping characteristics of aircrafts. Moreover, moving vehicles on bridge structure will result in a time-varying vehicle-bridge interaction problem which must be well solved in order to assure the operation safety of high-speed railway [2]. Nowadays time-varying characteristics of system parameters are often neglected in dynamic analysis of engineering structures which will lead to inaccurate results in the dynamic response prediction. In order to improve the accuracy of dynamic response prediction of time-varying structures, it is increasingly important to consider the time-varying features of dynamical systems.

At present, deterministic approaches are widely used to solve dynamic problems of time-varying structures. After the spatial discretization of structures to several elements, the problem is turned into an initial value problem of linear ordinary differential equations (ODEs) with time-varying coefficients. Then time integration methods, such as the central difference method, Newmark-beta method, can be used to solve these ODEs in time domain.

For transient analysis of time-varying structures, Penny and Howard [3] developed a time finite element method (TFEM) for a time-varying single degree-of-freedom system based on Hamilton's principle. Yu et al. [4] extended the TFEM to time-varying multiple degrees of freedom systems based on Hamilton's law of varying action. Zhao and Yu [5] presented a transient analysis method for linear time-varying structures based on multi-level sub-structuring method which improve the computational efficiency to a certain extent. For very complex systems in practical engineering, numerous number of elements are needed in deterministic approaches to describe the vibration behavior of time-varying structures, especially in high frequency range. The computation cost will increase rapidly to predict dynamic response of complex structures. Therefore, energy based methods, such as statistical energy analysis (SEA, [6]) and extended methods of SEA, are more suitable to analyse the problem.

The SEA, which is largely inspired from statistical physics [7], has been used to predict the average energy response of complex engineering systems for many years. Energy responses of subsystems can be calculated efficiently by solving energy transfer equations between subsystems in SEA. However, traditional SEA is an approach for steady state problems.

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Over the past several decades, some methods are proposed as extensions of the SEA to analyse unsteady state problem of time-invariant structures. SEA is firstly applied to predict transient energy response of time-invariant structures by transient statistical energy analysis (TSEA, [6]). Then, Lai and Soom [8,9] proposed the concept of "time-varying coupling loss factor" for time-invariant structures in TSEA, however, it is just a concept in mathematics and hard to interpret in physics. Furthermore, Pinnington and Lednik [10,11] applied TSEA to a two-oscillator system and a coupling-beam system, results from TSEA were compared with exact analytical solutions which showed that the prediction of TSEA is sometimes far from fitting the exact reference solution. Song et al. [12] applied affine arithmetic to TSEA of a two-oscillator system, and revealed the influence of measurement errors of parameters on predicted transient energy response. Experimentally, Robinson and Hopkins [13,14] used TSEA to calculate the maximum time-weighted sound and vibration levels in built-up structures. Good agreement was achieved between measurements and predictions. The transient local energy approach (TLEA), which was proposed by Ichchou et al. [15], is another extension of SEA to predict transient energy response of structures. With consideration of time-varying item in energy flow term, TLEA has much higher precision than TSEA [16].

The current research on dynamic response prediction of time-varying structures mostly concentrates on deterministic approaches, which are prohibitly time consuming for complex structures. Thus the statistical energy based approaches are preferred. On the other hand, though SEA has been developed at 1960s, research work mainly focused on transient energy prediction of time-invariant structures and literatures on prediction of transient energy response of time-varying structures are very limited. Therefore, by deriving the energy governing equations of time-varying structures, the SEA is firstly applied to predict transient energy response of time-varying structures in this paper.

The outline of this work is as follows: In Section 2, energy governing equations of time-varying structures are derived and SEA is applied to predict transient energy response of time-varying structures for the first time. In Sections 3 to 5, the effectiveness and accuracy of SEA method for time-varying system are verified by numerical examples, which include comparison with Newmark-beta method of a simply two-oscillator system under an impulse excitation, comparison with finite element method of a time-varying L-shaped folded plate with an initial energy, an application on a complex time-varying vibro-acoustic system under an impulse excitation. Finally, in Section 6, conclusions are drawn.

2. Statistical energy analysis for time-varying structures

Some basic definitions and assumption used to derive the energy equation of time-varying structure are generally summarized as [16]

$$e(s,t) = e^{+}(s,t) + e^{-}(s,t) \tag{1}$$

$$I(s,t) = I^{+}(s,t) + I^{-}(s,t) \tag{2}$$

$$I^{\pm}(s,t) = \pm c \cdot e^{\pm}(s,t) \tag{3}$$

where $e^{+}(s,t)$ and $e^{-}(s,t)$ are energy density associated with the right and left train waves, while $I^{+}(s,t)$ and $I^{-}(s,t)$are the incident and reflected power flows, the right train wave is considered separate from the left one. c is the energy velocity, the same as the group velocity of waves in a slight damping medium.

The local power balance for a non-loaded region can be expressed as

$$\frac{\partial e(s,t)}{\partial t} + P_{diss} + \frac{\partial I(s,t)}{\partial s} = 0 \tag{4}$$

The dissipation power for time-varying structures is evaluated

$$P_{\text{diss}} = \eta(t)\omega e(s,t) \tag{5}$$

where $\eta(t)$ is the time-varying damping loss factor (DLF), $\omega = 2\pi f$ is radian frequency.

Substituting Eqs. (1)–(3) and (5) into Eq. (4) yields:

$$c\frac{\partial e(s,t)}{\partial s} + \frac{1}{c}\frac{\partial I(s,t)}{\partial t} + \frac{\eta(t)\omega I(s,t)}{c} = 0 \tag{6}$$

Differentiating Eq. (4) with respect to time and space, respectively, leads to the expression

$$\frac{\partial^2 e(s,t)}{\partial t^2} + \eta(t)\omega\frac{\partial e(s,t)}{\partial t} + e(s,t)\omega\frac{\partial \eta(t)}{\partial t} + \frac{\partial^2 I(s,t)}{\partial s \partial t} = 0 \tag{7}$$

$$\frac{\partial^2 I(s,t)}{\partial s \partial t} + c^2\frac{\partial^2 e(s,t)}{\partial s^2} + \eta(t)\omega\frac{\partial I(s,t)}{\partial s} = 0 \tag{8}$$

By subtracting Eqs. (7) and (8) leads to the expression

$$\frac{\partial^2 e(s,t)}{\partial t^2} + \eta(t)\omega\frac{\partial e(s,t)}{\partial t} + e(s,t)\omega\frac{\partial \eta(t)}{\partial t} - c^2\frac{\partial^2 e(s,t)}{\partial s^2} - \eta(t)\omega\frac{\partial I(s,t)}{\partial s} = 0 \tag{9}$$

Substituting the expression of $\partial I(s,t)/\partial s$ in Eq. (4) into Eq. (9)

$$\frac{\partial^2 e(s,t)}{\partial t^2} + 2\eta(t)\omega\frac{\partial e(s,t)}{\partial t} + e(s,t)\omega\frac{\partial \eta(t)}{\partial t} - c^2\frac{\partial^2 e(s,t)}{\partial s^2} + (\eta(t)\omega)^2 e(s,t) = 0 \tag{10}$$

For time-varying structure consists of N subsystems, the concept of total energy rather than energy density turns the Eq. (10) into the form

$$\left[\frac{\partial^2 e(s,t)}{\partial t^2} + 2\eta(t)\omega\frac{\partial e(s,t)}{\partial t} + e(s,t)\omega\frac{\partial \eta(t)}{\partial t} - c^2\frac{\partial^2 e(s,t)}{\partial s^2} + (\eta(t)\omega)^2 e(s,t)\right] = P_i(t) \tag{11}$$

where $\int_V e_i(s,t) = E_i(t)$, $E_i(t)$ is time-varying energy stored in subsystem i, $P_i(t)$ is time-varying injected power into subsystem i and $\int_V \left(-\frac{c^2}{\eta(t)\omega}\nabla^2 e(s,t)\right)dV = \sum_{\substack{j=1,\\ j\neq i}}^{N} (\eta_{ij}(t)\omega E_i(t)) - \sum_{\substack{j=1,\\ j\neq i}}^{N} (\eta_{ji}(t)\omega E_j(t))$, $\eta_{ji}(t)$ is time-varying coupling loss factor (CLF) between subsystem j to i.

The power balance equation for subsystem i can be given by

$$\frac{1}{\eta_i(t)\omega}\frac{d^2 E_i(t)}{dt^2} + 2\frac{dE_i(t)}{dt} + \frac{E_i(t)}{\eta_i(t)}\frac{d\eta_i(t)}{dt} + \left(\eta_i(t) + \sum_{j=1, j\neq i}^{N}\eta_{ij}(t)\right)\omega E_i(t) - \sum_{j=1, j\neq i}^{N}\left(\eta_{ji}(t)\omega E_j(t)\right) = P_i(t) \tag{12}$$

The power balance equation for time-varying structure is driven by following second-order ODEs

$$\frac{1}{\omega\eta(t)}\frac{d^2\mathbf{E}(t)}{dt^2} + 2\frac{d\mathbf{E}(t)}{dt} + \frac{\mathbf{E}(t)}{\eta(t)}\frac{d\eta(t)}{dt} + \omega\eta(t)\mathbf{E}(t) = \mathbf{P}(t) \tag{13}$$

where $\mathbf{E}(t) = [E_1(t), E_2(t), \cdots E_N(t)]^{\mathrm{T}}$ is time-varying energy vector, $\boldsymbol{\eta}(t)$ is time-varying losing factor matrix of time-varying structure, $\mathbf{P}(t) = [P_1(t), P_2(t), \cdots P_N(t)]^{\mathrm{T}}$ is time-varying input power vector, the superscript T represents matrix transpose.