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Bond-associated deformation gradients for peridynamic correspondence model

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ABSTRACT

Non-ordinary state-based peridynamic correspondence material model is known to have issues with material instability, i.e. the existence of zero-energy modes, due to non-unique mapping between deformation states and force states via the conventional peridynamic deformation gradient. In this paper, an alternative approach in which the deformation gradient hence force state are computed specifically for each individual bond is proposed to eliminate the material instability. Bond-associated deformation gradient is calculated based on deformation states within an individual bond's proximity, termed here as the bond-associated family, rather than the whole family. This bond-associated deformation gradient can better represents the force state of each individual bond from the deformation states within its proximity, and hence inherently resolves issues of material instability in the conventional correspondence material model. Parametric study on bond-associated horizon size indicates that the optimal size should be no less than the material point's horizon size but smaller than two times of that value. Comparisons against reference solutions using finite element method establish the validity and accuracy of the proposed formulation.

1. Introduction

Among peridynamic models [1], the correspondence model [2] is very useful in that it allows the direct use of classical continuum material constitutive models within peridynamic theory. Continuum material models are naturally incorporated into the peridynamic framework via the counterpart measures, such as deformation gradient and first Piola–Kirchhoff stress tensor. However, the correspondence model suffers from some practical difficulties, such as non-invertibility [3]. This non-invertibility is a manifestation of material instability rather than merely an artifact of meshless discretization. It can be understood as existence of many possible deformations of a family that result in the same force state. As a consequence, there could be many possible deformation states of the entire body for a given loading. This has the practical effect of introducing zero-energy deformation modes to the model that need to be suppressed. Various remedies for zero-energy mode control are available in the literature, such as fictitious spring-force based methods [4,5] and stabilized field state based methods [6–8,3]. Although can be used to alleviate instabilities arising from zero-energy modes, these methods have their own issues and limitations, such as tedious parameter tuning and problematic stress oscillation. Most importantly, these methods do not provide resolution to the fundamental problem in the correspondence formulation leading to these zero-energy modes.

To fully take advantage of correspondence model in direct incorporation of continuum material constitutive relationships for nonlinear deformation and fracture modeling, there is still a strong need for effective zero-energy control schemes to be developed. In this paper, bond-associated deformation gradients are proposed to stabilize the conventional correspondence formulation to inherently resolve its material instability issue. This paper is organized as follows: Section 2 gives details on various deformation gradient definitions in both continuum theory and peridynamic theory. Following this, derivation of force state based on the proposed bond-associated deformation gradient is presented in Section 3. In Section 4, parametric study to obtain optimal bond-associated horizon size is performed. Discussions and conclusions are drawn in Section 5.

2. Deformation gradient

2.1. Deformation gradient in continuum mechanics

The deformation gradient is the fundamental measure of deformation in continuum mechanics. It maps line segments in the reference configuration into line segments (consisting of the same material points) in the current configuration.

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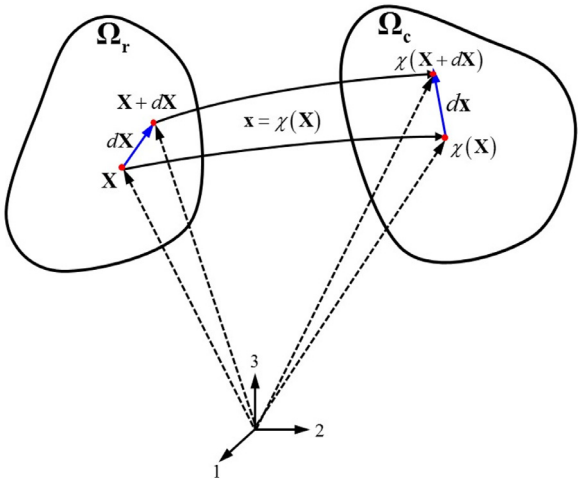

Fig. 1. Deformation of a continuum body.

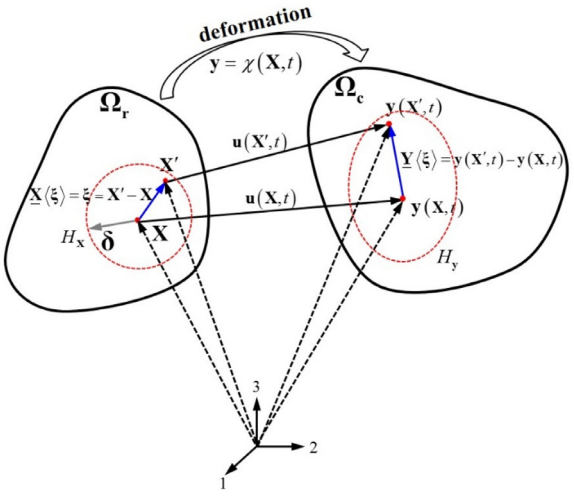

Fig. 2. Schematic illustrating different states in peridynamics.

Consider a line segment $d\mathbf{X}$ emanating from position $\mathbf{X}$ in the reference configuration Ω_r which deforms to $d\mathbf{x}$ in the current configuration Ω_c, see Fig. 1. Thus, the line segment in the deformed configuration Ω_c is given by

$$d\mathbf{x} = \chi(\mathbf{X}+d\mathbf{X}) - \chi(\mathbf{X}) \tag{1}$$

A Taylor expansion of $\chi(\mathbf{X}+d\mathbf{X})$ gives

$$\chi(\mathbf{X}+d\mathbf{X}) = \chi(\mathbf{X}) + \frac{\partial\chi}{\partial\mathbf{X}}(\mathbf{X})\cdot d\mathbf{X} + O(d\mathbf{X}) \tag{2}$$

where $O(d\mathbf{X})$ indicates higher-order terms of $d\mathbf{X}$.

Substituting Eq. (2) into Eq. (1) and assuming that $|d\mathbf{X}|$ is a infinitesimally small gives

$$d\mathbf{x} \approx \frac{\partial\chi}{\partial\mathbf{X}}(\mathbf{X})\cdot d\mathbf{X} \equiv \mathbf{F}(\mathbf{X})\cdot d\mathbf{X} \tag{3}$$

Eq. (3) tends to exact as the differential $d\mathbf{X}$ goes to zero.

The deformation gradient thus characterizes the deformation in the neighborhood of material point $\mathbf{X}$, mapping infinitesimal line segment $d\mathbf{X}$ emanating from $\mathbf{X}$ in the reference configuration to the infinitesimal line segment $d\mathbf{x}$ emanating from $\mathbf{x}$ in the deformed configuration.

2.2. Conventional peridynamic deformation gradient

Before formulating the peridynamic deformation gradient, some useful states represented in Fig. 2 are discussed below:

The *relative position vector state* of two material points in reference configuration Ω_r:

$$\underline{\mathbf{X}}\langle\xi\rangle = \xi = \mathbf{X}' - \mathbf{X} \tag{4}$$

where the angle bracket notation indicates that the state is associated with bond $\boldsymbol{\xi}$.

The *relative displacement vector state* of two material points:

$$\underline{\mathbf{U}}[\mathbf{X},t]\langle\xi\rangle = \eta = \mathbf{u}(\mathbf{X}',t) - \mathbf{u}(\mathbf{X},t) \tag{5}$$

where the square bracket notation has similar meaning to standard parentheses, indicating dependence on quantities, but is used for peridynamic states.

The *relative position vector state* or *deformation state* of two material points in the current configuration Ω_c:

$$\underline{\mathbf{Y}}[\mathbf{X},t]\langle\xi\rangle = \xi + \eta = \mathbf{y}(\mathbf{X}',t) - \mathbf{y}(\mathbf{X},t) \tag{6}$$

A finite distance that defines interactions between material points is called *horizon*. $H_\mathbf{X}$ denotes the horizon at material point $\mathbf{X}$with radius δ. For regular spatial discretization, the horizon size δ is usually represented in terms of mesh spacing Δx using a spacing factor m as

$$\delta = m\cdot\Delta x \tag{7}$$

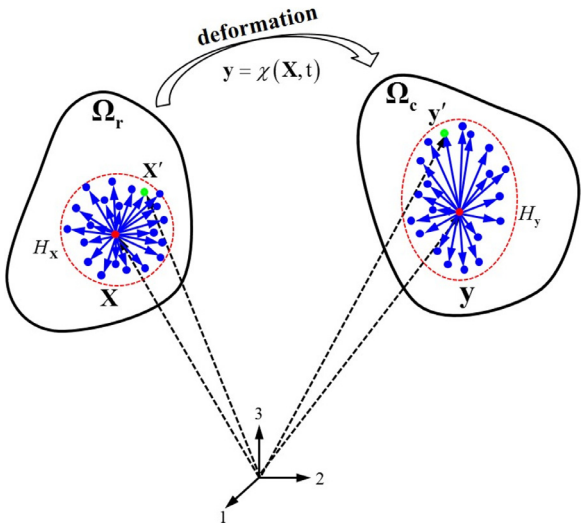

Fig. 3. Configuration for conventional peridynamic deformation gradient.

For a bond $\boldsymbol{\xi}$, there exists infinitely many mappings that transform the relative position vector state $\underline{\mathbf{X}}\langle\xi\rangle$ in the reference configuration to the relative position vector state $\underline{\mathbf{Y}}\langle\xi\rangle$ in the current configuration. A possible transformation can be written as:

$$\underline{\mathbf{Y}}\langle\xi\rangle = \mathbf{F}_\xi\cdot\underline{\mathbf{X}}\langle\xi\rangle \tag{8}$$

where $\mathbf{F}_{\boldsymbol{\xi}}$ is the deformation gradient for bond $\boldsymbol{\xi}$ connecting material point $\mathbf{X}$ and its neighboring material point $\mathbf{X}'$. Here, Eq. (8) defines the operation of a peridynamic deformation gradient in a manner analogous to the continuum deformation gradient in that it maps a vector state in the reference configuration to a vector state in the current configuration. $\mathbf{F}_{\boldsymbol{\xi}}$ becomes the deformation gradient at material point $\mathbf{X}$ when bond $\boldsymbol{\xi}$ tends to an infinitesimal length.

Writing Eq. (8) for material point $\mathbf{X}$ and each of the bonds connecting it with its neighbors leads to a system of over-constrained linear equations that cannot generally satisfied by a single mapping $\mathbf{F}_{\boldsymbol{\xi}}$. See Fig. 3 for reference. For this reason, a technique to compute an optimal deformation gradient $\mathbf{F}$ is sought. Techniques