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An adhesion model for plane-strain shearable hyperelastic beams

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ABSTRACT

In the present work, a new adhesion model is proposed to analyze the peeling behavior of plane-strain shearable hyperelastic beams from a rigid flat substrate. The large strain effect, the bending effect and the transverse shear effect are all taken into account in the model. The variational method is utilized to derive the equilibrium equations and associated boundary conditions, including one that physically means the local peeling (fracture) criterion. A first integral is found for such kind of beams and is also used to derive an equivalent global peeling criterion. It is proven that the critical peeling force for the steady peeling of such shearable hyperelastic beams is the same as that for hyperelastic thin films with membrane approximation. The effect of pre-stretch on the peeling behavior is further considered. The developed model will contribute to the modeling and understanding of the adhesion and fracture behaviors of soft structures and biomimetic adhesives.

1. Introduction

Detachment of thin, flexible films by peeling is a ubiquitous phenomenon of practical importance to a wide range of problems. Examples include the fabrication and reliability of multifunctional layered components [5,6], adhesive tapes used to fix objects in place [11,39,40], transfer printing of micro/nano-scale materials and devices from one substrate to another [9,37,41], the ability of plants and animals to cling to surfaces [4,24,30,34], and the achievement of physiological functions of tissues involving cell contact, adhesion and mechanotransduction [10,31,35].

Peeling mechanics of thin films has been extensively studied [2–4,7,8,12,18,19,21,26–30,33,34]. Many theoretical works were conducted via employing the inextensible elastica model [8,18,26] or extensible elastica model [17,19,27,28]. The finite rotation of the peeled film can be described by such kind of models, while inextensibility or small strain assumption is adopted in these models. For the case of small-angle peeling with a moderate interfacial adhesion energy or large-angle peeling with a strong interfacial adhesion energy, large strain probably occurs in the peeled film, and thus the elastica-based adhesion models fail to accurately describe the peeling behavior. The detachment of hyperelastic membranes from a flat substrate has also been studied by adopting the membrane approximation [2,7,12,38]. These adhesion models adopt hyperelastic constitutive relations and account for the large strain effect. However, the bending effect (bending resistance) is neglected in such types of models, which may be of great importance in some cases [18,19,27,34]. As far as we know, the bending effect and the large strain effect have not been simultaneously considered in a single theoretical adhesion model.

It is known that hyperelastic beam models account for the bending deformation, while most hyperelastic beam models do not consider the variation of the cross-section of the beam under large strain [1,36]. Thus the large strain effect is not accurately described in these models. In a recent work [15], we proposed a new finite strain beam model which accounts for the thickness stretchability (with the plane strain assumption). Hence, both the bending effect and the large strain effect are captured in this model. Based on this model, an adhesion model was developed to describe the peeling behavior of Euler-type hyperelastic beams [13]. However, the transverse shear effect is neglected in the adhesion model. It is known that for a moderately thick beam, the shear effect has a significant effect on the mechanical behavior of the beam. Motivated by such a gap, we will incorporate the shear effect into the adhesion model, and consequently, develop a new adhesion model for shearable hyperelastic beams within the plane-strain context.

The remainder of this paper is structured as follows. In Sections 2.1 and 2.2, the kinematics and constitutive relations for shearable hyperelastic beams are briefly presented. Based on these results and by using the variational method, the equilibrium equations and associated boundary conditions are derived in Section 2.3. In the subsequent subsection, a first integral is found for the equilibrium equation and it is then utilized to derive the global peeling criterion and also the critical force for steady peel-

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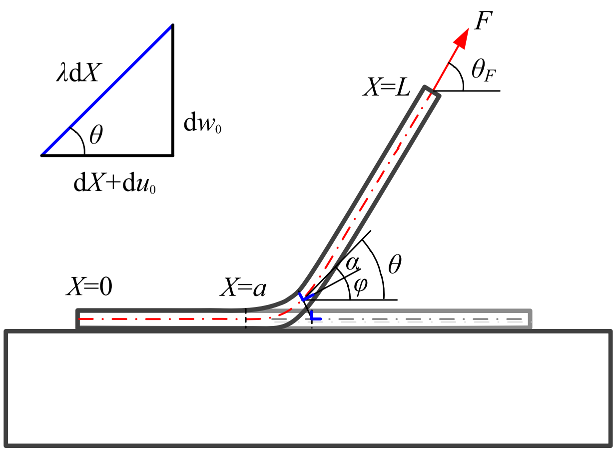

Fig. 1. Schematic figures for the reference and current configurations of a shearable hyperelastic beam lying on a rigid flat substrate and subjected to a peeling force.

ing. The pre-stretch effect is further considered in Section 2.5. At last, some conclusions are presented in Section 3.

2. Theoretical modeling

2.1. Kinematics

A finite strain beam model was proposed for plane-strain shearable hyperelastic beams in a previous work [14]. We will use the model to establish a new adhesion model to describe the peeling behavior of a plane-strain shearable hyperelastic beam. It is assumed in that model that any planar cross-section of the beam remains planar after deformation, and the beam is transversely shearable. However, the rigid cross-section hypothesis usually adopted in the classical Timoshenko beam model is relaxed by considering the thickness stretchability. Moreover, for the sake of simplicity, the plane strain assumption is adopted. Thus for a initially straight hyperelastic beam with rectangular cross-section (the width and thickness denoted by B and H, respectively) as shown in Fig. 1, deformation only occurs in the O-XZ plane. The plane strain assumption is applicable to beams with stiff fiber constraint in the width direction [16].

The deformation of a plane-strain hyperelastic beam from an initial stress-free configuration, which is referred to as the reference configuration, can be described by a mapping $\mathbf{x}=\boldsymbol{\chi}(\mathbf{X})$, *i.e.*, any material point denoted by its initial position $\mathbf{X}$ in the reference configuration is moved to a new position $\mathbf{x}$. In a Cartesian coordinate system, it can be written as $x=X+u(X, Z)$, $y=Y$, $z=Z+w(X, Z)$, where u and w are the horizontal and vertical components of the displacement of any material point in the beam, respectively. According to the aforementioned deformation hypothesis, we have the following expressions for the two displacement components:

$$u(X,Z) = u_0(X) - z^*(X,Z)sin[\varphi(X)],$$
$$w(X,Z) = w_0(X) + z^*(X,Z)\cos[\varphi(X)] - Z, \tag{1}$$

where u_0 and w_0 are the displacement components of any point on the geometrical mid plane, $z^* = \int_0^Z \lambda_Z \mathrm{d}Z$, in which λ_Z is the stretch of any line element dZ and the absolute value of z^* means the deformed distance between the material point (X, Y, Z) to the corresponding one (X, Y, 0) on the geometrical mid plane, and φ is the rotation angle of the cross-section. The slanted angle and stretch of any line element dX on the deformed geometrical mid plane are denoted by $\theta(X)$ and $\lambda(X)$, respectively. According to the geometric relation as shown in Fig. 1, it is easy to find that:

$$\theta = \arctan\frac{w_0'}{1+u'_0}, \lambda = \sqrt{\left(1+u_0'\right)^2 + w_0'^2}, \tag{2}$$

where ( )′ represents derivative with respect to the coordinate X.

Due to the shear deformation, the slanted angle θ of the tangent plane of the deformed geometrical mid plane is not the same as the cross-sectional rotation angle φ. The difference between them, denoted by $\alpha=\theta-\varphi$, is obviously the shear angle. The derivative φ' physically means the nominal bending curvature of the beam (not equal to the nominal curvature θ' of the deformed geometrical mid line) and it is denoted by a new symbol κ.

Through detailed kinematic analysis [14], it can be found for incompressible materials that:

$$\lambda_X = \sqrt{\lambda^2 - 2\kappa Z}. \tag{3}$$

$$\lambda_Z = \left(\lambda^2\cos^2\alpha - 2\kappa Z\right)^{-1/2}. \tag{4}$$

$$I_1 = \mathrm{tr}\mathbf{C} = \lambda^2 - 2\kappa Z + \left(\lambda^2\cos^2\alpha - 2\kappa Z\right)^{-1} + 1 \tag{5}$$

where λ_X is the stretch of any (initially horizontal) line element dX. It is noted that for the present homogeneous plane strain beam, we also have $I_2=I_1$, $I_3=1$, where $I_k(k=1, 2, 3)$ are the principal invariants of the right Cauchy-Green deformation tensor $\mathbf{C}=\mathbf{F}^{\mathrm{T}}\mathbf{F}$ with $\mathbf{F}=\partial\mathbf{x}/\partial\mathbf{X}$ the deformation gradient.

2.2. Constitutive equations

For the studied finite strain beam, the strain energy per unit reference length is defined by

$$\phi(\lambda,\alpha,\kappa) = \int_A W\mathrm{d}A, \tag{6}$$

where W is the strain energy per unit reference volume of the beam, and the area integral is over the referential (undeformed) cross-section of the beam. It is noted that Simo [36] derived the constitutive relations for spatial rods based on the 3D finite deformation theory and the assumed rigid planar cross-section hypothesis. Following Simo's method, and neglecting some additional strain energies due to thickness stretching, which are very small compared with the dominant stretching, shear and bending energies, we obtain the energy formula for shearable and thickness stretchable beams,

$$\delta\phi = N_n\delta\lambda_n + N_s\delta\gamma + M\delta\kappa, \tag{7}$$

which is the same with that given by Simo [36] and also by Ressiner [32]. Eq. (7) is equivalent to the following constitutive equations:

$$N_n = \frac{\partial\phi}{\partial\lambda_n}(\lambda,\alpha,\kappa),$$
$$N_s = \frac{\partial\phi}{\partial\gamma}(\lambda,\alpha,\kappa),$$
$$M = \frac{\partial\phi}{\partial\kappa}(\lambda,\alpha,\kappa), \tag{8}$$

where $\lambda_n=\lambda\cos\alpha-1$ and $\gamma=\lambda\sin\alpha$ are the normal strain and shear strain on the deformed planar cross-section (whose rotation angle is φ), and N_n, N_s and M are the normal stress resultant, shear stress resultant and bending moment, respectively, on the same cross-section. Eq. $(8)_{1-3}$ shows that the generalized forces N_n, N_s and M on the cross-section are respectively work conjugated to the corresponding generalized strains λ_n, γ and κ.

With the substitution of the expressions for λ_n and γ into Eq. (7), we also have:

$$\delta\phi = T\delta\lambda + S\lambda\delta\alpha + M\delta\kappa, \tag{9}$$