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Analytical modeling of nonlinear flexural-extensional vibration of flexure beams with an interconnected compliant element

Moeen Radgolchin[a], Hamid Moeenfard[a,b,*]

[a] School of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Khorasan Razavi, Iran
[b] Center of Excellence in Soft Computing and Intelligent Information Processing (SCIIP), Ferdowsi University of Mashhad, Mashhad, Khorasan Razavi, Iran

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ABSTRACT

The constraint behavior of compliant mechanisms can be improved via strengthening the stiffness of their constitutive beams using intermediate elements. This interior element may be assumed to be perfectly rigid or one can consider its compliance as a design parameter. While modeling the static behavior of such systems is state of art, the nonlinear dynamic of such systems have been remained un-investigated. So the objective of this paper is to suggest an analytical framework for modeling nonlinear free and forced vibrations of a simple flexure beam strengthened via a compliant intermediate element. The equations of motion of the system are derived using Hamilton's principle. Based on a single mode approximation, the partial differential equations of motion are transformed into two temporal equations. Employing multiple time-scales perturbation techniques, free vibration time histories and forced vibration response due to base excitation is derived analytically. Different parametric studies are carried out to recognize the effect of the intermediate compliant element on the vibrational behavior of the flexure beam. The results of this paper are expected to develop a new approach in modeling and investigation of load-displacement behavior of compliant mechanisms.

1. Introduction

The advent of compliant mechanisms in technological areas has created a new design generation for many precision engineering apparatuses. The performance of these mechanisms can be guaranteed somehow by their appropriate design and analysis. Compliant modules benefit from elastic deformation of mechanical elements to provide a guided motion. Compared to traditional mechanisms, implementation of compliant elements in mechanical units, facilitates the resulted structure with high motion resolution. The resulting motion is free of backlash, friction and undesired parasitic errors. Such superiorities extend the applications of compliant mechanisms to numerous areas including precision actuation instruments [4,30], precision hinges [10], energy harvesting devices [13], compliant grippers [12], multi-stable structures [5,18,26], micro/nano-manipulators [29], and so many other products (please see [8,9] for more examples).

A well-designed flexure unit should provide a large motion stroke in some specific directions (known as DoFs) along with negligible displacement along others (called the DoCs). Flexure beams are the core building block of most compliant mechanisms which convey the motion to the final stage via their elastic deformation. To have a large motion range, the constitutive flexure beams have to undergo large deflections which bring about nonlinearities due to arc length conservation, curvature, wrapping and trapeze effects [27]. Among these sources of nonlinearities, the conservation of arc length which is resulted from axial displacement of flexure beams sections, plays a significant role in dynamic behavior of such systems. This nonlinearity is the origin of the most important motion attributes of compliant mechanisms such as load-stiffening in the DoF and stiffness degradation in DoC directions. Using different design schemes for compliant mechanisms (such as parallel/serial configurations as well as different geometrical and mechanical specifications), their constraint behavior can be enhanced. As some examples, Yamakawa et al. [31] developed a six-DoF compliant structure enable of producing large highly-constrained planar motion. Yao et al. [32] proposed a hybrid assembly composed of serial-parallel flexure units which eliminate the undesired rotations of the motion stage. Brouwer et al. [3] employed a parallel kinematic design of flexure beams for micro-manipulations of MEMS devices. Hao and Kong [6] presented a 3-DoF compliant manipulator capable of providing large range decoupled motion. Although these efforts have developed countless design strategies

* Corresponding author at: Center of Excellence in Soft Computing and Intelligent Information Processing (SCIIP), Ferdowsi University of Mashhad, Mashhad, Khorasan Razavi, Iran.

E-mail address: h_moeenfard@um.ac.ir (H. Moeenfard).

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into the area of compliant mechanisms, the idea of improving the motion characteristics of flexure beam is another systematic scenario which can be implemented to any compliant unit. Based on this approach, an intermediate element is attached to flexure beams of a mechanism to alter the load-displacement behavior of the outcome. As a result, the undesired characteristics such as stiffness drop or load-stiffening effects are alleviated. This idea was first developed in [27] where an interconnected rigid segment was introduced into the mathematical model of a flexure beam. The result of this manipulated design was enhancing the constraint behavior of flexure beams [1]. This achievement was also spread to more complex mechanisms by replacing the constitutive elements with rigidly reinforced counterparts [28]. As some examples, by taking advantages of this newly-proposed model, double parallelogram (DP), DPDP, tilted DPDP and clamped DPDP compliant mechanisms were upgraded to improve the travelling range of com-drive actuators [19–21].

The major un-addressed question about the reinforcement strategy is to determine a criterion for the concept of perfectly rigid reinforcement. If so, one needs to find out whether the rigid reinforcement idea brings about the best load-displacement results. To answer these questions, recently, a new more accurate formulation has been presented in which the compliance of the intermediate element was taken into account in the load-displacement relations of simple flexure units [23]. The proposed formulation was proved to more accurately simulate the nonlinear constraint behavior of basic compliant modules such as flexure beams and parallelogram mechanisms.

While different attributes of various compliant mechanisms have been well-addressed in prior arts, the problem of dynamic behavior of flexure units is an important issue which has not been yet considered properly. Such a study is complicated however, due to the multi-body nature of the mechanism as well as the activation of geometrically nonlinear behavior. To overcome such difficulties, the complex mechanisms need to be sub-divided into elements whose dynamic can be studied much more easily. As it was mentioned earlier, flexure beams are the main building block of many compliant modules and can be considered as the key to disclose dynamic of complex structures. While the dynamic modeling of beams in different conditions are well-developed in the literature ([7,17,22]), flexure beams have not yet been addressed properly. Among few examples dealt with investigation of the dynamic behavior of flexure elements, one can mention the work of Moeenfard and Awtar [14]. In their research, the nonlinear vibration of flexure beams carrying a tip mass was considered. Here in this paper, the nonlinear flexural-extensional vibrations of flexure beams enhanced via a compliant interconnected element are simulated analytically. Using the exact mode shapes of the system, the time history of the free vibration and nonlinear frequency response of the forced vibration of the beam are derived and the effect of reinforcement properties on these responses is investigated. It will be demonstrated that the compliance of the intermediate body plays an important role in the dynamic specifications of the system.

2. Problem formulation

The Schematic view of a typical flexure beam with length L is shown in Fig. 1. A tip mass M is attached at the end of the beam and the structure is subjected to a base excitation $Y_b(\hat{t}) = Y_0 \sin\omega\hat{t}$. For the sake of generality, the flexure beam is assumed to be consisted of a not necessarily rigid intermediate reinforcement with length $L-2a$. The height and thickness of the beam's primary (unreinforced) sections are denoted as H and T respectively.

Compliant mechanisms are made up of long slender beams which are excited in the mid-range frequency. In such circumstances, the beams can be accurately modeled in the context of the Euler-Bernoulli beam theory [15,25]. Based on this theory, plane cross sections remain plane and normal to the neutral axis. On the other hand, flexure mechanisms impose large transverse deflections to its beams up to 10–15% of their length. At these large deformations, a linear theory is no longer capable of describing the beam's behavior and geometric nonlinearities come into play. Accordingly, the nonlinear axial strain of an element located at a distance Y from the neutral axis will be as

$$\varepsilon_{XX} = \frac{\partial U}{\partial X} + \frac{1}{2}\left(\frac{\partial W}{\partial X}\right)^2 - Y\left(\frac{\partial^2 W}{\partial X^2}\right)^2 \tag{1}$$

where U and W are the axial and transverse displacement components respectively.

Assuming the flexure beam to be composed of linear elastic materials, the total strain energy of the system would be stated as

$$V = \frac{1}{2}\iiint_{\Gamma} E\varepsilon_{XX}^2 d\Gamma \tag{2}$$

In the above relation, Γ is the volume and E denotes the Young's modulus of elasticity of the beam's material. Inserting the strain expression (1) into (2), assuming constant material properties along the beam and performing some mathematical manipulations, the overall strain energy of the beam can be written as $V = \sum_{i=1}^{6} V_i$, where [23]

$$V_1 = \frac{EA}{2a}\left\{U_a(t) + \frac{1}{2}\int_0^a \left(\frac{\partial W}{\partial X}\right)^2 dX\right\}^2 \tag{3}$$

$$V_2 = \frac{EI}{2}\int_0^a \left(\frac{\partial^2 W}{\partial X^2}\right)^2 dX \tag{4}$$

$$V_3 = \frac{E_i A_i}{2(L-2a)}\left\{U_{L-a}(t) - U_a(t) + \frac{1}{2}\int_a^{L-a} \left(\frac{\partial W}{\partial X}\right)^2 dX\right\}^2 \tag{5}$$

$$V_4 = \frac{E_i I_i}{2}\int_a^{L-a} \left(\frac{\partial^2 W}{\partial X^2}\right)^2 dX \tag{6}$$

$$V_5 = \frac{EA}{2a}\left\{U_L(t) - U_{L-a}(t) + \frac{1}{2}\int_{L-a}^{L} \left(\frac{\partial W}{\partial X}\right)^2 dX\right\}^2 \tag{7}$$

$$V_6 = \frac{EI}{2}\int_{L-a}^{L} \left(\frac{\partial^2 W}{\partial X^2}\right)^2 dX \tag{8}$$

In these equations, I and A respectively represent the second area moment of inertia and area cross sections of the primary beam sections and I_i and A_i are the corresponding values for the intermediate section. Moreover, $U_a(t)$, $U_{L-a}(t)$ and $U_L(t)$ are the axial displacements at $X=a$, $X=L-a$ and $X=L$ respectively.

The radius of gyration of the flexure beam under study is considered to be very small. In such condition, the axial inertia becomes negligible [16]. So the kinetic energy of the flexure beam and its tip mass can be states as $T = \sum_{i=1}^{4} T_i$, in which

$$T_1 = \frac{\rho A}{2}\int_0^a \left(\frac{\partial}{\partial t}\left(Y_b(\hat{t}) + W(X,\hat{t})\right)\right)^2 dX \tag{9}$$

$$T_2 = \frac{\rho_i A_i}{2}\int_a^{L-a} \left(\frac{\partial}{\partial t}\left(Y_b(\hat{t}) + W(X,\hat{t})\right)\right)^2 dX \tag{10}$$