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Strong discontinuity analysis of a class of anisotropic continuum damage constitutive models – Part I: Theoretical considerations

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ABSTRACT

Several modeling techniques aiming at considering cracks as kinematics discontinuities have been proposed for the past years. Within this scope, the embedded finite element method (E-FEM) was introduced a couple of years ago. Among the features of this approach, it has been shown that a kinematic enhancement of the displacement field allows constructing a discrete model (expressed in terms of traction vector–displacement jump) from any continuous model (expressed in terms of stress–strain). This result has been rigorously established if the continuous model is formulated within the framework of either isotropic continuum damage or plasticity theories. The objectives of this study are (i) to extend this result in case where the continuous model belongs to a class of anisotropic continuum damage constitutive models and (ii) to show the main features of a specific traction/separation law derived from the aforementioned class of constitutive models through several numerical case-studies. In this paper, the light is put on the theoretical considerations which allow deriving discrete models in a consistent way.

1. Introduction

Cracking is one of the major contributing factors to the energy dissipation when dealing with failure analysis of quasi-brittle materials. For the past decades, many authors focused on developing consistent approaches to model this dissipative mechanism.

The existing approaches can be sorted in two categories, depending on the fact if cracks are modeled in a localized way or in a diffuse way. The first category includes constitutive laws expressed within the theoretical frameworks of continuum damage mechanics and plasticity. However, it is well known that mesoscopic quantities, such as crack openings or spacings, cannot be predicted accurately because of the diffuse description of cracking. In order to overcome this drawback, alternative approaches based upon a kinematics enhancement of the displacement field came up. In this second category, we can distinguish two subgroups of techniques, according to the type of kinematics enhancement considered. Some of them assume a local[1] enhancement [1–3] whereas others postulate a global[2] [4,5].

Among the approaches based upon a local kinematics enhancement, the strong discontinuity method (SDM) is one of the most used [6,7] because of (i) its low-intrusiveness in computational softwares and (ii) the absence of additional degrees of freedom, keeping unchanged the structure of the algebraic system of equations to be solved out. Within this framework, it has been shown that the kinematics enhancement of the displacement field allows defining a discrete constitutive law (expressed in terms of traction vector and displacement jump), from specific classes of continuous constitutive laws (expressed in terms of stress and strain) [8–10]. Especially, this result has been rigorously established in case of a continuous constitutive model expressed within the framework of either isotropic continuum damage mechanics or plasticity theories.

The objective of this paper is to show that a strong discontinuity kinematics enhancement leads naturally to the definition of

[1] The term *local* refers to the fact that the additional degrees of freedom related to the kinematics enhancement can be condensed at the finite element level. The size of the algebraic system of equations to be solved is not modified with respect to the case of a classical kinematics.

[2] The term *global* refers to the fact that the additional degrees of freedom are considered at the node.

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a discrete constitutive model even though a family of anisotropic continuum damage based constitutive models is considered. In other words, it is proposed to extend the result shown by [8] to a class of anisotropic continuum constitutive models derived from micromechanical assumptions. While the existence of a closed-form expression of the effective stress tensor $\underline{\underline{\tilde{\sigma}}}$ is straightforward in case of isotropic continuum damage theory, this feature is no more ensured when dealing with anisotropy. Indeed, the multiplicative structure of the stress tensor $\underline{\underline{\sigma}} = (1-d)\underline{\underline{\tilde{\sigma}}}$, d standing for the scalar damage variable, plays an important role when deriving the underlying discrete model, as shown by [8].

In order to achieve this objective, the paper is outlined as follows. Firstly, the theoretical framework defining the class of continuum models considered in this study is exposed. Secondly, the strong discontinuity analysis of this class of constitutive models is presented. It aims at analyzing the kinematics compatibility of the continuum damage based framework with the unbounded strain field, derived from the displacement field enhanced with a discontinuous part. Thirdly, the discrete constitutive framework which can be naturally derived from the aforementioned analysis is presented.

2. Anisotropic continuum damage mechanics based framework

2.1. State potential

Among the various ways to express the state potential, the one retained in this study lies in choosing the Helmholtz free energy. It is expressed as follows:

$$\Psi(\underline{\underline{\varepsilon}}, (\rho_i)_{i=1,\cdots,n}) = \Psi_0(\underline{\underline{\varepsilon}}) - \Psi_a((\rho_i)_{i=1,\ldots,n}, \underline{\underline{\varepsilon}}) \tag{1}$$

where Ψ is the Helmholtz free energy, $(\rho_i)_{i=1,\ldots,n} \geq 0$ are directional internal variables that may be interpreted as micro-cracking densities in the directions $i = 1, \ldots, n$, $\Psi_0(\underline{\underline{\varepsilon}}) = \frac{1}{2}\underline{\underline{\varepsilon}} : \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}} \geq 0$ is the elastic contribution of the Helmholtz free energy and $\Psi_a((\rho_i)_{i=1,\ldots,n}, \underline{\underline{\varepsilon}}) \geq 0$ the inelastic one. It is important to notice that the part of the energy stored due to hardening process has not been included in Eq. (1) for the sake of simplicity. It is assumed that Ψ_a is expressed as follows:

$$\Psi_a((\rho_i)_{i=1,\ldots,n}, \underline{\underline{\varepsilon}}) = \sum_{i=1}^{n} \rho_i g_i(\underline{\underline{\varepsilon}}) \tag{2}$$

where $(g_i)_{i=1,\ldots,n} : \mathcal{E} \to \mathbb{R}^+$, $\mathcal{E}$ being the space of compatible strain tensors. $(g_i)_{i=1,\ldots,n}$ are assumed to be combinations of quadratic strain-based terms in order to ensure (i) the convexity and (ii) the continuity of the second order derivative of the state potential. The functions g_i can be chosen according to tensorial representation theories in order to particularize the way of taking into account the damage anisotropy [11–13]. Furthermore, Eq. (1) shows that the Helmholtz free energy is progressively decreased to zero when a dissipative process involving the flow of the micro-cracking density variables is activated. The first derivatives of the set of functions $(g_i)_{i=1,\ldots,n}$ are assumed to be linear combinations of strain-based terms. It is interesting to notice that the isotropic case can be recovered if: (i) all the damage density variables are chosen such as they have similar flow rules i.e. $\rho_i = \rho_0, \forall i \in \{1, \ldots, n\}$ and (ii) the terms $\frac{dg_i}{d\underline{\underline{\varepsilon}}}$ are chosen such as $\sum_{i=1}^{n} \frac{dg_i}{d\underline{\underline{\varepsilon}}}(\underline{\underline{\varepsilon}}) = \underline{\underline{\sigma}}_0(\underline{\underline{\varepsilon}})$.

2.2. State equations

The general expressions of the state equations can be obtained by differentiating the state potential with respect to the state and internal variables. The Cauchy stress tensor can be expressed as follows:

$$\underline{\underline{\sigma}}(\underline{\underline{\varepsilon}}) = \frac{\partial \Psi}{\partial \underline{\underline{\varepsilon}}}(\underline{\underline{\varepsilon}}, (\rho_i)_{i=1,\cdots,n}) = \underline{\underline{\sigma}}_0(\underline{\underline{\varepsilon}}) - \sum_{i=1}^{n} \rho_i \frac{dg_i}{d\underline{\underline{\varepsilon}}}(\underline{\underline{\varepsilon}}) \tag{3}$$

where $\underline{\underline{\sigma}}_0 = \underline{\underline{\underline{\underline{C}}}} : \underline{\underline{\varepsilon}}$ is the elastic contribution to the Cauchy stress tensor. One can notice that the Cauchy's stress $\underline{\underline{\sigma}}$ is progressively decreased when the micro-cracking density variables flow. The remaining state laws lie in defining the n thermodynamic forces related to the micro-cracking density variables. They are expressed as follows:

$$F_{\rho_i}(\underline{\underline{\varepsilon}}) = -\frac{\partial \Psi}{\partial \rho_i}(\underline{\underline{\varepsilon}}) = g_i(\underline{\underline{\varepsilon}}) \tag{4}$$

2.3. Flow rules

The micro-cracking density variables are assumed to be independent.[3] Therefore, n independent threshold surfaces are introduced in order to manage the flow of each variable ρ_i. They can be expressed as follows:

$$\phi_{\rho_i}(F_{\rho_i}, Z_i) = F_{\rho_i} - Z_i \tag{5}$$

where $Z_i \in [0, Z_0]$ is the thermodynamic variable related to a negative isotropic hardening mechanism, allowing to describe the softening response in the inelastic regime. More precisely, the variable Z_i can be expressed through the definition of a consolidation function H_i (function of the hardening variable z_i) as follows:

$$Z_i = \frac{dH_i(z_i)}{dz_i} \tag{6}$$

Assuming associative flow rules, the rate of the micro-cracking density variables ρ_i can be expressed as follows:

$$\dot{\rho}_i = \dot{\lambda}_i \frac{\partial \phi_{\rho_i}}{\partial F_{\rho_i}} = \dot{\lambda}_i \tag{7}$$

where $(\dot{.})$ stands for the rate of $(.)$ and $\dot{\lambda}_i$ is the Lagrange multiplier related to the i^{th} dissipative mechanism. The internal variable z_i, related to the thermodynamic force Z_i, can also be managed by assuming an associative flow rule:

$$\dot{z}_i = \dot{\lambda}_i \frac{\partial \phi_{\rho_i}}{\partial Z_i} = -\dot{\lambda}_i \tag{8}$$

One can notice that variables z_i and ρ_i are flow-coupled because $\dot{\rho}_i = -\dot{z}_i$. Finally, the following loading/unloading conditions should be satisfied:

$$\dot{\lambda}_i \geq 0; \quad \phi_{\rho_i} \leq 0; \quad \dot{\lambda}_i \phi_{\rho_i} = 0 \tag{9}$$

3. Strong discontinuity analysis

3.1. Enhanced kinematics

Let us denote by Ω a body in $\mathbb{R}^3$. Let us also consider that a strong discontinuity occurs at a certain location Γ in Ω, such as Ω is split into two sub-bodies Ω^+ and Ω^-. These two sub-bodies can be identified in a consistent manner according to the normal $\underline{n}$ to the discontinuity Γ (line in 2D or a plane in 3D). The discontinuous displacement field can be expressed as follows:

$$\underline{u}(\underline{x}, t) = \underline{\overline{u}}(\underline{x}, t) + \mathcal{H}_\Gamma(\underline{x})[\underline{u}](\underline{x}, t) \tag{10}$$

[3] The term *independent* means there is no flow-coupling.