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Mesoscale bounds in viscoelasticity of random composites

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ABSTRACT

Under consideration is the problem of size and response of the representative volume element (RVE) of spatially random linear viscoelastic materials. The model microstructure adopted here is the random checkerboard with one phase elastic and another viscoelastic, perfectly bonded everywhere. The method relies on the hierarchies of mesoscale bounds of relaxation moduli and creep compliances (Huet, 1995, 1999) obtained via solutions of two stochastic initial boundary value problems, respectively, under uniform kinematic and uniform stress boundary conditions. In general, the microscale viscoelasticity introduces larger discrepancy in the hierarchy of mesoscale bounds compared to elasticity, and this discrepancy grows as the time increases.

1. Introduction

The prediction of macroscopic properties of randomly structured heterogeneous materials is of major importance in many engineering applications. Evidently, any material displays heterogeneity on a micro scale and has properties depending on the scale of approximating continuum. The key issue, which commonly arises when dealing with structure–property relations of such materials is the validity of *separation of scales* of the continuum mechanical model

$$\left.\begin{matrix} d \ll \\ d < \end{matrix}\right\} L \ll L_{macro}. \tag{1}$$

Here L is the size of the so-called *representative volume element* (RVE), d is the microscale, and L_{macro} is the macroscale. The RVE is clearly set up in two basic cases [6]: (i) a unit cell in a periodic microstructure, and (ii) a domain containing infinitely many microscale elements (e.g. inclusions) in a randomly structured medium.

This paper develops scale-dependent bounds on the effective response of random composites for a linear viscoelastic material with a two-phase, spatially random composition: a planar random checkerboard microstructure, in which one phase is viscoelastic and another elastic, with perfect bonding everywhere. We follow the approach that originated with [2,10,11]: we deal with *mesoscale* (or *apparent*) material properties of finite size domains subjected to uniform boundary conditions of either kinematic or traction types. In general, these properties are random and mesoscale-dependent, and hence, such a domain is a *statistical volume element* (SVE). As L/d increases, the SVE tends towards the RVE – case (ii) above. Of interest is the evaluation of this trend from SVE to RVE in the time domain; the frequency domain properties of this model problem for a full range of volume fractions will be dealt with in a separate and more extensive paper. The SVE-to-RVE scaling issue has already been studied in many different settings besides linear elastic: conductivity, physically nonlinear elasticity, finite (thermo)elasticity, elasto-plasticity, permeability [6,8]. A theoretical basis for the viscoelastic setting was developed by Huet [3,4] and this forms a stepping-stone for the present work, see also [5].

2. Problem formulation

2.1. Random microstructure

The random material is taken as a set of all the realizations $B(\omega)$ parametrized by sample events ω of the Ω space

$$\mathcal{B} = \{B(\omega); \omega \in \Omega\}. \tag{2}$$

Any realization $B(\omega)$ of the composite $\mathcal{B} = \{B(\omega); \omega \in \Omega\}$, while spatially disordered (i.e., heterogeneous), follows deterministic laws of mechanics.

The spatial (volume-type) averages will be denoted by the overbar , while the statistical (or ensemble) averages by $\langle \cdot \rangle$. That is, if

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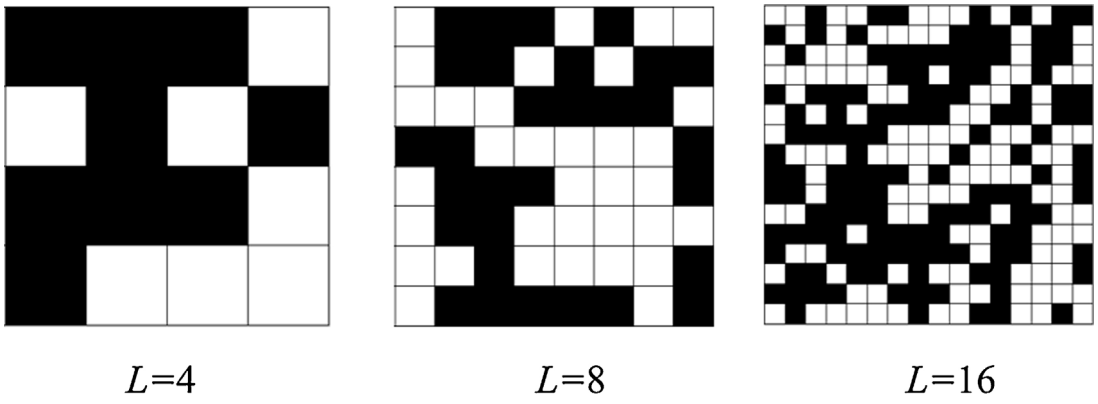

Fig. 1. Sample realizations of the random, two-phase checkerboard on there $L \times L$ lattices.

we have a random (n-component, real valued) field $\boldsymbol{\Theta}$ defined over some probability space $\{\Omega, F, P\}$ (with F being a σ-field and P a probability measure) and over some domain V in $\mathbb{R}^2$ of volume V

$$\boldsymbol{\Theta} : \Omega \times X \to \mathbb{R}^n, \tag{3}$$

the said averages are

$$\overline{\boldsymbol{\Theta}(\omega)} \equiv \frac{1}{V}\int_v \boldsymbol{\Theta}(\omega, \mathbf{x})dV \quad \langle \boldsymbol{\Theta}(\mathbf{x}) \rangle \equiv \int_\Omega \boldsymbol{\Theta}(\omega, \mathbf{x})dP. \tag{4}$$

As the random material we take a so-called *random chessboard* (or *checkerboard*) in two-dimensions, where each square cell of M sites is occupied, independently of realizations at all other cells, with probability p_1 and p_2 by phases 1 and 2, respectively. Clearly, for a square lattice $L \times L = M$, the number of different realizations is $|\Omega| = 2^{L\times L}$. Given the construction process, each ω occurs with probability $1/2^{L\times L}$. Technically speaking, it is a Bernoulli lattice process with the probability $p = 1/2$. Fig. 1 shows sample realizations at three different scales and a nominal volume fraction 50%. Indeed, the latter is the numerical setting in our computational mechanics reported below.

In the following, it will be convenient to work with a mesoscale, a nondimensional parameter

$$\delta = \frac{L}{d} \quad (d = \text{size of one site}) \tag{5}$$

in the range $[0, \infty)$, so that $\mathcal{B}_{L/d}$, a *mesodomain*, will be written $\mathcal{B}_\delta$, etc. Thus, $\delta = 0$ signifies the pointwise description of the material, while $\delta \to \infty$ is the RVE limit. The *mesoscale random material* is a set of all the realizations $B_\delta(\omega)$ parametrized by sample events ω of the Ω space

$$\mathcal{B}_\delta = \{B_\delta(\omega); \omega \in \Omega\}. \tag{6}$$

2.2. Governing equations

First, recall that the constitutive equations of linear viscoelastic solids in the time domain are expressed in terms of temporal Stieltjes convolutions:

$$\sigma_{ij}(t) = \int_0^t r_{ijkl}(t-\tau) : \dot{\epsilon_{kl}}(\tau)d\tau, \tag{7}$$

$$\epsilon_{ij}(t) = \int_0^t f_{ijkl}(t-\tau) : \dot{\sigma_{kl}}(\tau)d\tau, \tag{8}$$

where an overdot denotes the derivative with respect to time, while the tensors $r_{ijkl}(t)$ and $f_{ijkl}(t)$ are the *relaxation modulus* and *creep compliance*, respectively. With $\circ$ indicating the convolution integral, Eqs. (7) and (8) can be expressed in a simplified indicial form:

$$\sigma_{ij} = r_{ijkl} \circ \epsilon_{kl}, \tag{9}$$

$$\epsilon_{ij} = f_{ijkl} \circ \sigma_{kl}. \tag{10}$$

Linear viscoelasticity of our random material is implemented using the Prony series

$$g_R(t) = 1 - \sum_{i=1}^{N} \overline{g_i}(1 - e^{-(t/\tau_i)}), \tag{11}$$

where $g_R(t)$ is the normalized shear modulus which starts from 1 at $t = 0$ and then gradually decreases. $\overline{g_i}$ and τ_i are parameters that can be fitted to resemble the performance of a real relaxation behavior. For illustrative purposes in the present study, the linear viscoelasticity is assigned to one phase only and only one term in (11) is considered – this corresponds to a generalized Maxwell (i.e., Zener) model. The other phase is kept elastic. There is perfect bonding everywhere. Properties of both phases are given in Table 1. The relaxation moduli of both phases are also plotted in Fig. 2. From the figure, it can be concluded that the generated composite is like half rigid elastic solids mixed with half relatively "soft" viscoelastic gel, with the solids' modulus twice that of the gel.

Table 1
Properties of two phases.

	Type	E	ν	g_1	τ_1
Mat A	Elastic	60	0.3		
Mat B	Viscoelastic	30	0.3	0.9	0.25

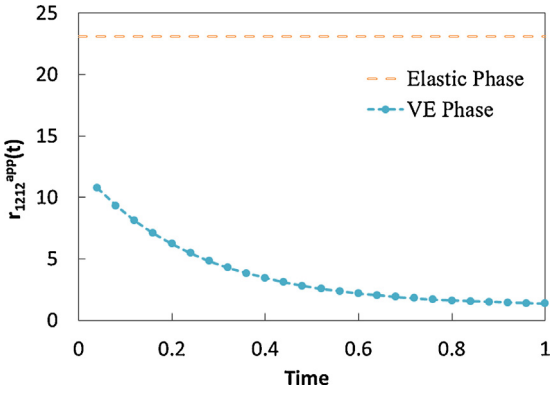

Fig. 2. Relaxation moduli of both component phases.