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Use of equivalent body forces for acoustic emission from a crack in a plate

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ABSTRACT

A method to determine acoustic emission of surface waves from a crack near the free edge of a plate, is presented, in terms of the function $f(t)$, which defines the time dependence of the crack opening process, the crack opening volume per unit thickness of the plate, and the elastic constants of the plate. The determination of the time-varying displacement is based on the use of equivalent body forces, which are shown to be two double forces. The acoustic emission of the crack, or the equivalent radiation from the double forces, has been obtained by a novel use of the elastodynamic reciprocity theorem. It is of interest that the normal surface-wave displacement at a position x_0 of the free edge comes out as depending on df/dt evaluated at x_0 for $t > x_0/c_R$, where c_R is the velocity of surface waves on the free edge.

1. Introduction

Acoustic emission (AE) is the wave motion produced in a solid body by damage processes, particularly by the opening of a crack in a stressed body. For the opening of a crack the emanating acoustic emission can provide information on the location of the crack and possibly also on its size. For that reason there is a continuing interest in the measurement and interpretation of acoustic emission. For a few recent contributions we refer to Okafor and Natarajan [1] and Zhao et al. [2].

The interpretation of AE requires a measurement model based on a theoretical or numerical analysis of the ultrasonic effects from generation to measurement. A complete analysis of that kind is very complicated due to the variety of effects that may play a role. Clearly it is important that the generation by the opening crack is described in as simple of a manner as possible.

In this paper we consider the two-dimensional plane stress problem of acoustic emission of surface waves from a through crack near the free edge of a thin plate. The crack, of length l, is located in a plane normal to the free edge. The acoustic emission actually includes body waves and surface waves, but at some distance the body waves have attenuated and only the surface waves are detectable.

To simplify the analysis, it is shown that for wave lengths that are larger than the largest characteristic dimension of the crack we can define equivalent concentrated body forces, which when applied in the absence of the crack, produce the same radiation as is produced by the actual opening of the crack. The system of equivalent body forces consist of two double forces in the plane of the plate. The magnitudes of these double forces are in terms of elastic constants and the crack-opening volume of the crack per unit thickness of the plate, produced in the process of acoustic emission.

Analytical expressions for the acoustic emission are obtained by a novel method based on the elastodynamic reciprocity theorem. This theorem is formulated for two states, A and B, where state A is the actual radiation from the concentrated double forces and state B is a suitable virtual wave. This method has been discussed in some detail by Achenbach [3].

2. Acoustic emission

The two-dimensional configuration of the crack near a free surface is shown in Fig. 1. The free surface is defined by $z=0$ in a Cartesian (x,z) coordinate system. The plate of thickness d is thin, and a state of plane stress defined by $\partial/\partial y \equiv 0$ and $\tau_{yy}=0$ is considered. This implies that the usual elastic constant λ must be replace by $\bar{\lambda}$, where

$$\bar{\lambda} = \frac{2\mu\lambda}{\lambda + 2\mu} = \frac{1-2\nu}{1-\nu}\lambda \qquad (1)$$

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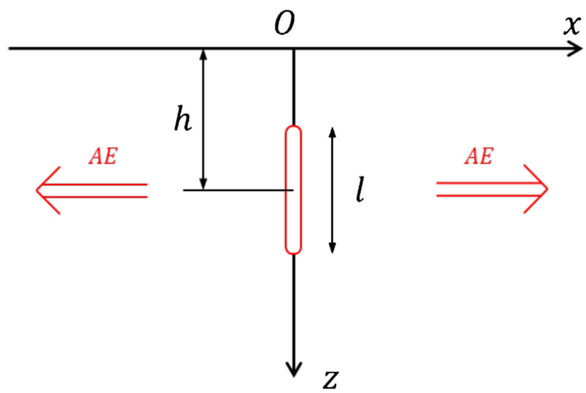

Fig. 1. Acoustic emission from a sub-surface crack.

where ν is Poisson's ratio. The acoustic emission of surface waves is indicated by *AE*.

Prior to crack formation the surface of the impending crack can be thought of as being loaded by tensile stresses $\tau_{xx}(0,z)$ generated by the loading *F*, which keep the crack closed as if there were no crack. At time $t=0$ these tensile stresses are decreasing with time dependence $f(t)$.

The crack which is located in the plane $x=0$ opens up in Mode I, over a time defined by the function $f(t)$. The crack opening displacements over the length of the crack, *l*, may be defined by

$$u_x\left(x^-,z\right)f(t)=-u_x\left(x^+,z\right)f(t) \tag{2}$$

It is convenient to convert $f(t)$ to frequency dependence by the exponential Fourier transform

$$\hat{f}(\omega)=\int_0^\infty f(t)e^{+i\omega t}\mathrm{d}t \tag{3}$$

and

$$f(t)=\frac{1}{\pi}Re\int_0^\infty \hat{f}(\omega)e^{-i\omega t}\mathrm{d}\omega \tag{4}$$

The corresponding time-harmonic displacements on the crack faces may then be written as

$$u_x\left(x^-,z\right)\hat{f}(\omega)e^{-i\omega t}=-u_x\left(x^+,z\right)\hat{f}(\omega)e^{-i\omega t} \tag{5}$$

We temporarily omit $\hat{f}(\omega)$ and employ displacements as

$$u_x\left(x^-,z\right)e^{-i\omega t}=-u\left(x^+,z\right)e^{-i\omega t} \tag{6}$$

The crack opening displacement (COD) is

$$\begin{aligned}\mathrm{COD}&=\left[u_x\left(x^+,z\right)-u_x\left(x^-,z\right)\right]e^{-i\omega t}=2u_x\left(x^+,z\right)e^{-i\omega t}\\&=\Delta u_x(0,z)e^{-i\omega t}\end{aligned} \tag{7}$$

The crack opening volume (COV) per unit plate thickness follows as

$$\Delta V_x e^{-i\omega t}=\int_l \Delta u_x(0,z)\,\mathrm{d}z e^{-i\omega t} \tag{8}$$

here *l* is the length of the crack.

For further convenience the term $\exp(-i\omega t)$ is also temporarily omitted in the sequel.

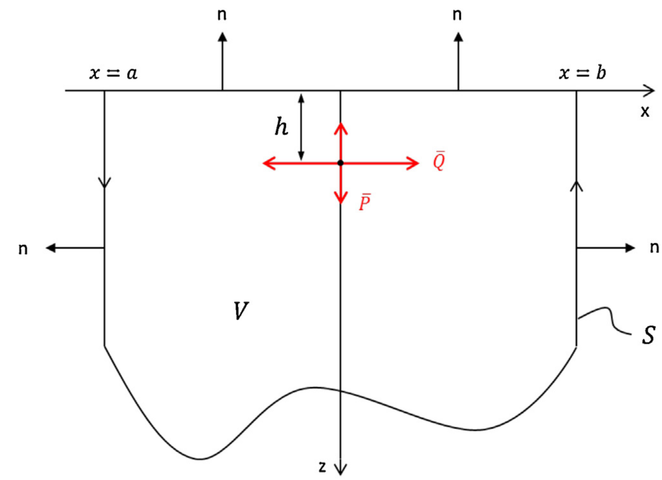

Fig. 2. *V* with boundary *S* for application of the elastodynamic reciprocity theorem.

3. Equivalent body forces

In the acoustic emission process the crack can be represented by a system of body forces which, when applied in the absence of the crack, will produce the same radiation as the acoustic emission from the crack. Such body forces are called equivalent body forces. For more general configurations such equivalent body forces have been derived in earthquake seismology, by Burridge and Knopoff [4] and for cracks in structures by Rice [5]. They come out in the form of double forces or dipoles. For the present geometry they have been derived in the usual forms in a somewhat simplified manner.

The acoustic emission is considered a field that has been generated by a strain discontinuity which over the length *l* may be written as

$$\varepsilon^D(0,z)=\Delta u_x(x,z)\delta(x)e^{-i\omega t} \tag{9}$$

where $\Delta u_x(0,z)$ is defined by Eq. (7). The quasi-static equations of motion for plane stress are used to determine the corresponding body forces *P* and *Q*:

$$\frac{\partial\tau_{xx}^D}{\partial x}+\frac{\partial\tau_{xz}^D}{\partial z}+Q=0 \tag{10}$$

$$\frac{\partial\tau_{zx}^D}{\partial x}+\frac{\partial\tau_{zz}^D}{\partial z}+P=0 \tag{11}$$

where the superscript indicates stresses due to the strain discontinuity, and

$$\tau_{xx}^D=\left(\bar{\lambda}+2\mu\right)\Delta u_x(x,z)\delta(x) \tag{12}$$

$$\tau_{zz}^D=\bar{\lambda}\Delta u_x(x,z)\delta(x) \tag{13}$$

The stress τ_{zx}^D is not discontinuous. Next we integrate along the length of the crack to obtain

$$\bar{\tau}_{xx}^D=\left(\bar{\lambda}+2\mu\right)\Delta V_x\delta(x)\delta(z-h) \tag{14}$$

$$\bar{\tau}_{zz}^D=\bar{\lambda}\Delta V_x\delta(x)\delta(z-h) \tag{15}$$

These integrated stresses are now integrated quantities centered at $z=h$. From Eqs. (10) and (11) we then obtain, as shown in Fig. 2:

$$\bar{Q}=-\left(\bar{\lambda}+2\mu\right)\Delta V_x\delta'(x)\delta(z-h) \tag{16}$$

$$\bar{P}=-\bar{\lambda}\Delta V_x\delta(x)\delta'(z-h) \tag{17}$$

where $\bar{P}$ and $\bar{Q}$ are body forces per unit plate thickness, integrated over the length of the crack. In Eqs. (16) and (17) the derivative of the delta function indicates a double force.

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