Mechanism and Machine Theory 129 (2018) 106–130

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Nonlinear behaviour design using the kinematic singularity of a general type of double-slider four-bar linkage

Baokun Li[a,b], Guangbo Hao[b,*]

[a] School of Mechanical Engineering, Anhui University of Science and Technology, Huainan 232001, China
[b] School of Engineering, University College Cork, Cork T12K8AF, Ireland

ARTICLE INFO

Article history:
Received 13 February 2018
Revised 9 May 2018
Accepted 24 July 2018

Keywords:
Nonlinear behaviour
Kinematic singularity
Linkage with springs
Force-displacement formulation

ABSTRACT

Kinematic singularities existing in rigid-body linkages are applied to design the mechanism with nonlinear (stiffness) behaviours. The kinematic singularities of a general type of double-slider four-bar linkage, whose two paths of the sliders are not perpendicular, are identified by analysing the kinematic formulation. A new type of mechanism is synthesized by placing translational springs and torsional springs at corresponding joints. Two motion models are proposed to design the different types of nonlinear behaviours. After constructing the force-displacement formulation of the mechanism, four types of nonlinear behaviours including the bistable behaviour, the partial negative stiffness behaviour, the partial zero-stiffness behaviour and the positive stiffness behaviour are investigated. The influences of designed parameters including spring stiffness, geometry parameters, and initial input position on nonlinear characteristics are illustrated. The results indicate that the mechanism exhibits corresponding nonlinear behaviours with these different parameters. A method of generating an expected nonlinear behaviour over a range of input displacement is developed, with a particular emphasis on the constant-force behaviour. The approach of designing the nonlinear behaviour based on the rigid-body linkages with springs expands the application range of the kinematic singularities of rigid-body linkages, and can also be employed to construct nonlinear compliant mechanisms.

1. Introduction

Kinematic singularity, one of intrinsic properties of a mechanism, may decrease the stability, disable the motion ability or change the degree of freedom of the mechanism [1]. As kinematic singularity seriously affects the performance of the mechanism, many different methods are proposed to investigate the kinematic singularity classification, singularity identification and singularity property, with a particular emphasis on eliminating or avoiding the singularities [2–8].

However, kinematic singularity of the mechanism can also be employed to create new devices. A force sensor was designed using the characteristic that actuation forces increase greatly when the parallel manipulator works near the kinematic singular configuration [9]. Since a compliant mechanism usually works around a given position for a small range of motion, a rigid-body mechanism is often used to design a compliant mechanism based on the pseudo-rigid-body model [10,11]. Some new types of compliant mechanisms are synthesized by using this approach when the rigid-body mechanism works around the kinematic singular configuration with special property [12–16]. A type of reconfigurable mechanism was advanced by

* Corresponding author.
E-mail addresses: bkli@aust.edu.cn (B. Li), G.Hao@ucc.ie (G. Hao).

https://doi.org/10.1016/j.mechmachtheory.2018.07.016
0094-114X/

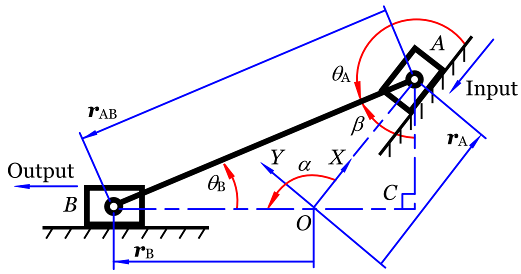

Fig. 1. Double-slider four-bar linkage.

employing the characteristic that the kinematic property changes when the mechanism works in kinematic singularity [12]. A compliant reconfigurable gripper was constructed by replacing the joints with flexures based on the equal-length four-bar linkage when the crank and the rocker work in one line [13]. When a parallel mechanism works in the kinematic actuation-singularity, it will gain at least one extra degree of freedom, which was applied to synthesize a new type of compliant medical device [14,15]. Kinematic singularity can also be used to provide the desired nonlinear behaviours. Based on a kinematic formulation of the necessary condition in presence of parallel singularities of the four-bar rigid-body mechanism, a nonlinear softening spring is found in [16].

Another type of mechanism with nonlinear behaviour is the bistable compliant mechanism constructed by using the post-buckling of compliant beams [17–21], or compliant mechanisms designed by applying the rigid-body replacement approach [22,23]. Reference [22] illustrated that the double-slider four-bar compliant mechanism could be used to produce the bistable behaviour. The double-slider four-bar compliant mechanism, with two paths of the sliders being vertical, was used to design the nonlinear stiffness characteristics of compliant mechanisms [23]. However, for a general type of double-slider linkage with springs, its force-displacement formulation has not been established and comprehensively analysed. Moreover, the method of designing a mechanism with an expected nonlinear characteristic has not been explored yet. With a similar motivation to design nonlinear mechanisms [22,23], this paper will investigate the nonlinear behaviour design using the kinematic singularity property of a generic double-slider four-bar linkage with springs. The rest of the paper is organised as follows: in Section 2, after constructing the kinematic formulation of a general type of double-slider four-bar linkage, kinematic singularity of the linkage is identified and analysed. In Section 3, the force-displacement formulation of the mechanism considering stiffness of springs at joints is deduced using the principle of virtual work. Section 4 introduces the mechanism may generate four types of nonlinear behaviour, and offers the method of producing the corresponding nonlinear behaviour, as well as discusses the influences of different parameters on nonlinear behaviour of the mechanism. Section 5 presents the further discussion followed by conclusions drawn in Section 6.

2. Kinematics and kinematic singularities

The schematic of a general type of double slider four-bar linkage is shown in Fig. 1. Without loss of generality, the input slider moves along the negative direction of *X*-axis. In order to derive the kinematic formulae of the linkage, the Cartesian coordinates system *O-XYZ* is attached on the base. The direction of *Z*-axis is determined by the right hand rule, the origin *O* is the intersection point of all axes, and α is the rotation angle from the positive direction of *X*-axis to the initial moving direction of output slider. Point *C* is the intersection point of two perpendicular lines *OB* and *AC*. A vector equation of the linkage can be written as

$$\boldsymbol{r}_{\mathrm{A}} + \boldsymbol{r}_{\mathrm{AB}} = \boldsymbol{r}_{\mathrm{B}}, \tag{1}$$

where vector $\boldsymbol{r}_{\mathrm{A}}$ and vector $\boldsymbol{r}_{\mathrm{B}}$ are the position vectors of points *A* and *B* with respect to the coordinates system *O-XYZ*, respectively, and vector $\boldsymbol{r}_{\mathrm{AB}}$ is the vector from point *A* to point *B* with respect to the frame *O-XYZ*.

We can transform Eq. (1) into two algebraic equations as follows

$$\begin{cases} r_{\mathrm{A}} + r_{\mathrm{AB}} \cos\theta_{\mathrm{A}} = r_{\mathrm{B}} \cos\alpha \\ r_{\mathrm{AB}} \sin\theta_{\mathrm{A}} = r_{\mathrm{B}} \sin\alpha \end{cases}, \tag{2}$$

where, scalar r_{A} is the coordinate of point *A* on *X*-axis, θ_{A} is the rotation angle from the positive direction of *X*-axis to coupler *AB*, scalar r_{B} is the distance between origin *O* and point *B*, and scalar r_{AB} is the length of the coupler *AB*, which is invariant during motion.

Considering the symmetry, in this paper α is limited to the following condition

$$0 < \alpha < 180^\circ. \tag{3}$$

The case that α is greater than 180° can be treated as the case of α less than 180°, which is explained in Appendix A.

The solutions of Eq. (2) for r_{B} with eliminating θ_{A} yields

$$r_{\mathrm{B}} = \sqrt{r_{\mathrm{AB}}^2 - r_{\mathrm{A}}^2 \sin^2\alpha} + r_{\mathrm{A}} \cos\alpha. \tag{4a}$$