Mechanism and Machine Theory 129 (2018) 175–190

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Design of convex variable radius drum mechanisms

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ARTICLE INFO

Article history:
Received 1 December 2017
Revised 14 June 2018
Accepted 22 July 2018

Keywords:
Variable radius drum mechanism
Wrapping cam
Convexity
Interval analysis
Static balancing
Cable-driven mechanisms

ABSTRACT

Variable radius drum (VRD) mechanisms represent an interesting alternative to more conventional transmission mechanisms whereby highly flexible input-output relationships may be achieved. The design of such mechanisms implies the determination of the VRD's profile based on performance criteria whose evaluation generally assumes continuous contact between the cable and the VRD. Although it has been previously acknowledged that this assumption is only valid if the VRD is convex, this condition has not yet have been verified. This deficiency is addressed herein by proposing a design approach which takes into account the requirement of VRD convexity. The approach is based on the formulation of a constrained minimization problem where a convexity constraint, evaluated using interval analysis techniques to ensure its satisfaction throughout the VRD's operating range, is applied. The proposed method is quite general and may be used in the design of VRDs based on various performance requirements. The effectiveness of the proposed approach is validated through its use in the design of VRDs for the static balancing of a one-degree-of-freedom robot arm as well as a one-degree-of-freedom parallel cable-driven mechanism. The method is demonstrated to be successful in producing VRD mechanisms that meet specified design criteria.

1. Introduction

A *variable radius drum*[1] (VRD) is understood here to represent a generalization of the more conventional constant radius drum (CRD), which consists of a cylindrically-shaped body that pivots about its longitudinal axes and on which a length of cable may be wound. A VRD mechanism, for its part, is defined as any mechanism that includes at least one VRD [1]. The purpose of VRD mechanisms is to convert translational displacements and/or forces to angular rotations and/or torques (or vice-versa). In this sense, VRD mechanisms may be viewed as alternatives to more conventional transmission mechanisms (*e.g.*, cam-follower systems, crank-slider mechanisms, etc.).

The use of VRD mechanisms has already been investigated somewhat extensively. A typical objective of their use is to transform the behavior of a linear extension spring so as to emulate a torsion spring with desired characteristics. Applications of this concept include the production of constant torques in shape memory alloy actuators [2], the static balancing of robots [3–5], the improvement of torque capacity in pneumatic artificial muscle actuators while maintaining their operating range [6,7] and the generation of desired torque-deflection profiles [8–10] including in series elastic actuators [11]. Another interesting application of VRD mechanisms targeted the conversion of an extension spring's force into constant contact forces between the wheels of a pipe inspection robot and the interior surface of the pipe [12]. Finally, VRD mecha-

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[1] These is also sometimes referred to as a *wrapping cam*.

https://doi.org/10.1016/j.mechmachtheory.2018.07.014

nisms were more recently used in the development of cable-driven robots that do not depend on gravity to maintain tension in their cables yet do not require redundant actuation [1,13]. In this case, the VRD mechanisms that were employed were tasked with converting angular rotations to desired changes in the lengths of the cables.

The design of a VRD mechanism typically amounts to the determination of the required profile (*i.e.*, drum radii as a function of angular location around its periphery) in order to meet design objectives. This has previously been achieved through a graphical approach combined with a numerical solver where a set of discrete points on the VRD profile are generated at specified angular increments such that a desired torque-deflection relationship is obtained [2,3,6,7]. For the case of the static balancing of a point mass attached to the end of a pivoted link [5] where simplifying assumptions were made, it was shown that the design of a VRD mechanism could be obtained from the solution of an ordinary differential equation. Meanwhile, several prior works have developed analytical closed-form solutions to obtain VRD profiles addressing various design requirements [1,4,8–10] based on different kinematic and static modeling approaches. In most cases, such methods involve the sequential use of several connected equations to generate a set of discrete points along the VRD profile. Finally, an optimization of discrete points along a VRD profile was also used in [11] where the objective was the least-squares minimization of differences between desired and actual torques in different mechanism configurations.

Previous works dealing with the design of VRD mechanisms have only used the parameters describing the VRD profile as design variables. In other words, the geometrical and mechanical parameters describing the remainder of the VRD mechanism (*e.g.*, link lengths, positions of anchor points, spring stiffness, etc.) have been treated as constants. In instances where unsatisfactory VRD profiles were obtained, however, these parameters were changed *a posteriori* based on design intuition until a suitable design was obtained.

An important design criteria for a VRD is to ensure that its profile is convex. If this criteria is not satisfied, the cable will not always be in contact with the VRD as it is wrapped around its profile and the VRD mechanism's predicted performance will be inaccurate. The issue of convexity in the design of VRDs has been acknowledged in previous works [4,8,9]. However, to the author's knowledge, no prior work has included the convexity of VRDs as a constraint during the design process or even as a post-design verification.

The work presented herein seeks to address the above mentioned deficiencies. The design of VRD mechanisms is achieved through a constrained minimization problem which allows the designer to (i) select a performance metric arbitrarily according to the design objective and (ii) include as many parameters (related to the VRD profile and/or the mechanism in which it is located) as design variables as desired. Moreover, the design methodology guarantees the convexity of the VRD throughout its range of operation by the application of a design constraint.

The paper is organized as follows. In Section 2, the proposed design methodology, including the formulation of the VRD convexity constraint, is presented in general terms. The use of the methodology is then illustrated by applying it to the design of VRD mechanisms to statically balance a one-degree-of-freedom (1-DoF) robot arm (Section 3) and to obtain a 1-DoF parallel cable-driven mechanism (PCDM) (Section 4). Finally, discussions and conclusions are presented in Section 5.

2. Description of the proposed VRD design methodology

As previously mentioned, the proposed design methodology for VRD mechanisms is based on a constrained minimization problem. In what follows, the details of this approach are presented in general terms such that it can be applied to any VRD mechanism design scenario.

2.1. Mathematical representation of the VRD profile

For the purpose of the methodology proposed herein, the VRD profile is defined in polar coordinates by a parametric equation $r(\phi)$ with coefficients $\mathbf{c} = [c_0, c_1, \dots]^T$ where, as illustrated in Fig. 1, $r(\phi)$ is the radius of the VRD at an angle ϕ along its profile. The type of parametric equation used (*e.g.*, polynomial, Fourier series, etc.) must be chosen at the beginning of the design process as the coefficients in $\mathbf{c}$ correspond to design variables. It is noted that $r(\phi)$ needs only to be defined over the VRD's operating range, *i.e.*, $\phi_{\min} \le \phi \le \phi_{\max}$. Outside this range, the VRD profile may be chosen somewhat arbitrarily based on secondary objectives (*e.g.*, ease of manufacture and assembly, etc.). As will later be seen, the VRD profile $r(\phi)$ must also be twice differentiable with respect to ϕ (*i.e.*, $dr/d\phi$ and $d^2r/d\phi^2$ must exist throughout the VRD's operating range) for the purpose of the convexity verification. The fact that the type of equation defining $r(\phi)$ is imposed *a priori* will generally prevent an exact solution to the VRD mechanism design problem from being found (*i.e.*, one that perfectly meets stipulated performance requirements). However, this is considered an acceptable compromise given the added benefit of verifying the convexity of the VRD profile as well as allowing the simultaneous optimization of other VRD mechanism parameters. In addition, as will be demonstrated, the proposed method has the ability to generate designs that satisfy performance requirements within tight tolerances.

2.2. Formulation of the VRD design optimization problem

The VRD mechanism design is formulated as an optimization problem seeking to minimize $\eta(\mathbf{x})$ subject to $\mathbf{g}(\mathbf{x}) \le \mathbf{0}$ where $\eta(\mathbf{x})$ is the objective function, $\mathbf{g}(\mathbf{x})$ is a vector of nonlinear inequality constraints and $\mathbf{0}$ is the zero vector. The vector of