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Research paper

A method based on the vanishing of self-motion manifolds to determine the collision-free workspace of redundant robots

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ABSTRACT

It is well known that there exist interior barriers within the boundaries of the workspace of redundant robots. These interior barriers, which are drastically affected by kinematic constraints, are very important for trajectory planning since they imply motion impediments for the robot. Existing geometrical and singularity-based methods that obtain such interior barriers cannot accommodate complex (yet common) kinematic constraints, such as the condition that collisions between different links should be forbidden. This paper presents a new sampling method to obtain the boundaries and interior barriers of the workspace of redundant robots considering collision constraints. The proposed method identifies the occurrence of barriers with the vanishing of connected components of self-motion manifolds. Our method consists of three phases: (1) densely sampling self-motion manifolds by solving the inverse kinematics, (2) clustering phase to identify disjoint self-motion manifolds, and (3) matching phase to detect the vanishing of self-motion manifolds. The presented method is illustrated with several examples involving redundant parallel robots, considering joint limits and collisions. These examples demonstrate the feasibility and usefulness of the proposed method, and illustrate the drastic changes suffered by workspace barriers due to collision constraints.

1. Introduction

This paper presents a method to obtain the boundaries and interior barriers of the workspace of redundant robots, under joint limits and collision constraints. We assume that the kinematic chain of the robot (which can be serial, parallel, or hybrid) has n degrees of freedom (DOF) and the dimension of its task space is m. This means that n actuated joint coordinates $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_n]$ are used to control m task variables $\mathbf{t} = [t_1, \ldots, t_m]$. Typically, θ_j are the lengths of linear actuators or the rotated angles of revolute actuators, whereas t_i define the position and/or orientation of some output link of the robot which is of interest for a given task. The workspace can be defined as the set of values that vector $\mathbf{t}$ can attain subject to the kinematics of the robot (i.e., the mathematical relationship between $\boldsymbol{\theta}$ and $\mathbf{t}$) and to other kinematic constraints (e.g., joint limits or avoidance of collisions).

When $m < n$, it is said that the robot is *kinematically redundant*[1] [23], which means that more DOFs than necessary are used to perform the task. Fig. 1 shows an example of redundant robot: a serial 3R planar robot in which $n = 3$ actuated

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[1] In the following, we will omit the word “kinematically”, assuming that all the redundancies mentioned in this paper are of this type.

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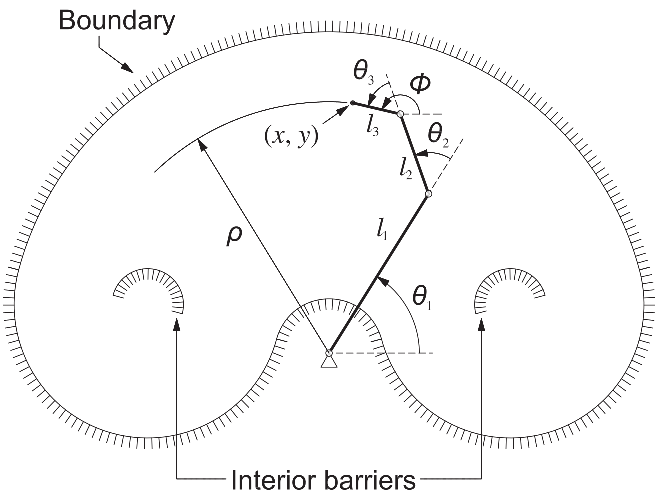

Fig. 1. Workspace of a redundant 3R serial robot with $l_1 = 17.3,\ l_2 = 7.8$ and $l_3 = 4.5$. The first joint angle is subject to joint limits $\theta_1 \in [15,\ 165]^\circ$; the second and third joints can freely rotate. This example is very similar to an example presented in [8].

revolute joints control the Cartesian coordinates $\mathbf{t} = [t_1, t_2] = [x, y]$ of its tip in the plane ($m = 2$). This robot becomes non-redundant if we also intend to control the orientation ϕ of link l_3 (which would add another task variable: $t_3 = \phi$).

Redundancy occurs when specifically designing robots with more DOFs than task variables, but also when studying some types of workspaces. For example, the workspace of a 6-DOF robot is the 6D set of spatial translations and orientations attainable by its end-effector. Since 6D sets cannot be represented, other 3D representations are necessary to visually analyze the workspace. One may fix the orientation of the end-effector and represent only the attainable translations, obtaining the constant-orientation workspace. Alternatively, one may project the 6D workspace to the 3D subspace of translations, obtaining the reachable workspace (i.e., the set of positions attainable by the end-effector with at least one orientation). Reachable workspaces, which are widely studied, are examples of "redundant workspaces", because they only retain information related to the translation of the end-effector, omitting its orientation.

Given the importance of the workspace of robots in their design and in motion planning, its calculation is a mature topic that has received much attention during the last decades. Many methods have been proposed, most of them can be classified into three groups [30]: geometrical methods, discretization/sampling methods, and singularity-based methods.

Geometrical methods are exact and very fast, but their scope is limited since they are tailored to specific classes of robots. These methods usually rely on Computer Aided Design tools and can be used to compute constant-orientation workspaces of parallel robots as the intersection of the workspaces of all limbs of the robot [4], which are simple geometric shapes such as solid tori [22], annuli [7,32], or spherical shells [17]. Geometric constructions have also been used to obtain analytic descriptions of the boundaries of reachable and other workspaces of parallel robots [3,32], as well as maximal singularity-free areas [20]. Geometrical methods can handle kinematic constraints such as joint limits and even self-collisions, but only in relatively simple cases [29].

Sampling methods generate many configurations of the robot, and check if each configuration belongs to the workspace satisfying all kinematic constraints. Thus, these methods can easily handle complex kinematic constraints, because one simply needs to check if all constraints are satisfied for each concrete configuration *after generating it* [5]. Configurations are typically generated by sampling points from the joint or task spaces, following regular or random patterns [Monte Carlo methods [5,18]]. For serial robots, configurations are usually generated by sampling points from the joint space and solving afterwards the Forward Kinematics (FK) [FK-based methods [11,37]], which is simpler than the Inverse Kinematics (IK) for these robots. Conversely, for parallel robots the IK is simpler than the FK. Thus, their configurations are often generated by sampling points from the task space and solving the IK for each point [IK-based methods [13,28,39]]. When using IK-based methods with redundant robots, since their inverse kinematic problem admits infinitely many solutions for a given task point, it is sufficient to find only one solution satisfying all constraints to guarantee that the considered task point belongs to the workspace [34].

Singularity-based methods directly obtain the boundaries delimiting the workspace, both for redundant and non-redundant robots. At these boundaries, $\mathbf{\Phi_z}$ becomes rank-deficient, where $\mathbf{\Phi_z}$ is a Jacobian matrix of derivatives of all kinematic constraints with respect to all variables involved in the problem (excluding task variables $\mathbf{t}$). The condition of $\mathbf{\Phi_z}$ being rank-deficient yields a system $\mathcal{S}$ of equations whose solution set contains the boundaries of the workspace. System $\mathcal{S}$ may be analytically solved in some cases [1,2], but in general it must be solved numerically [8,19]. In [19], $\mathcal{S}$ is solved via continuation, obtaining planar slices of workspace boundaries. Bohigas et al. [8] identified problematic situations in which continuation methods fail, such as degenerate boundaries and multicomponent workspaces. Alternatively, Bohigas et al. [8] manage to rewrite $\mathcal{S}$ as a quadratic system and solve it using a linear relaxation method which is robust to