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Research paper

Power losses in two-degrees-of-freedom planetary gear trains: A critical analysis of Radzimovsky's formulas

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ABSTRACT

Two-degrees-of-freedom gearing are key elements of any planetary gear train. Thus, the full understanding of their basic mechanics and mechanical efficiency evaluation is not only of significant theoretical interest, but important in many industrial applications. Some previous investigations on the estimation of mechanical efficiency in two-degrees-of-freedom differential gearing rely upon the relationships deduced by Radzimovsky. However, to the best of authors' knowledge, a critical analysis on the validity conditions of these relationships is not available yet in technical literature. To fill the apparent gap, in this paper a broad physical interpretation of mechanical efficiency analysis of two-dof planetary gear trains is offered. The discussion allowed to outline the theoretical limits of the Radzimovsky formulas. The current approach provides three different results for the power loss of a two-input gear pair entity with the planet carrier as the output link. This leads to the conclusion that even for the same PGT and for the same input and output links the power loss has, for each valid sequence of angular velocities, a peculiar mathematical expression.

1. Introduction

Modern hybrid gear drives can be considered as an assembly of two-dof planetary gear trains (PGT). As it will be herein discussed, the basic mechanics of a two-dof basic PGT may show not obvious features that limit the validity of classic formulas for computing its mechanical efficiency.

Apparently, the earliest contribution on the computation of mechanical efficiency in two dof PGT is reported in classic and often cited companion papers authored by Radzimovsky [1,2]. In this paper, through a critical analysis of Radzimovsky's formulas, their limitations will be outlined and explained.

A conventional approach for calculating the mechanical efficiency[1] of a two-dof GPE is based on the assumption that the gear train is a combination of two one-dof inversions of the gear train (e.g. [1–8]). However, as it will be herein shown, under certain circumstances the relationship among the powers flowing in a PGT cannot be treated as the sum of two separate powers flowing through the links of a gear drive having one of its driving links fixed.

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[1] In the present analysis we will assume members with constant velocities and meshing losses only. Friction among teeth will be also constant.

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Nomenclature

PGT:	Planetary gear train
One-dof:	One-degree-of-freedom
Two-dof:	Two-degrees-of-freedom
GPE:	Gear pair entity
link-i and link-j:	Meshing gears
link-k:	Gear carrier
$L_{i(j-k)}$:	Power loss of the GPE when link-i is fixed and links j and k are input and output links, respectively.
$L_{(i,j-k)}$:	Power loss of the GPE when operating with links i and j as input and k as output
$P_x = T_x\omega_x$:	Power through link-x. If $P_x > 0$, the link-x is an input or driving link. Conversely, If $P_x < 0$, the link-x is an output or driven link.
$P_x^y = T_x(\omega_x - \omega_y)$:	Potential or virtual power of link x when link y is considered fixed
$N_{j,i} = \pm\dfrac{Z_j}{Z_i}$:	Planet gear ratio (+: internally meshing gears, −: externally meshing gears)
T_x:	Torque on link x
Z_x:	Number of teeth on gear x
ω_x:	Absolute angular velocity of link x
η_E:	Planetary (Epicyclic) gear train mechanical efficiency
$\eta_{x(y-z)}$:	Mechanical efficiency of the GPE when link x is fixed and links y and z are driving and driven links, respectively.
$\eta_{(x,y-z)}$:	Mechanical efficiency of the GPE when operating with links x and y as driving links and z as driven.
$\eta_{(x-y,z)}$:	Mechanical efficiency of the GPE when operating with links y and z as driven links and x as driving link.

Pennestrì and Freudenstein [4], in their systematic method for mechanical efficiency analysis of planetary gear trains, combined the formulas which were used for estimating the efficiency of one-dof planetary gear train inversions deduced by Merritt [9] and Macmillan [10] with the Radzimovsky's formulas for two-dof differential gearing.

With regard to the efficiency of two-dof planetary gear trains, Pennestrì and Valentini [5] compared several gear train mechanical efficiency formulas. In the same reference was offered an interpretation of the analytical relations originally deduced by Radzimovsky [1,2] based on the general power balance equation of two parallel machine units.

Pennestrì et al. [6] expanded the formulas proposed by Pennestrì and Freudenstein [4], and applied them to power split transmissions for hybrid vehicles.

Mantriota and Pennestrì [11] proposed and experimentally validated load dependent losses model for epicyclic gear trains. Coaccioli and Pennestrì [8] evaluated the mechanical efficiency of an epicyclic gear drive in hybrid scooters based on graph algorithm.

The use of kinematic inversion in epicyclic gear train analysis has a well-established tradition. For instance, with reference to velocity analysis, in the tabular method the angular speeds of the links are obtained in two steps. First, the gear carrier (or arm) is assumed fixed. Then the relative velocity ratios are set equal to gear ratios, as in ordinary gear trains. The resulting system of equations is solved for the unknown angular velocities.

Merritt [9,12] has been the first to recognize the usefulness of kinematic inversion to obtain the mechanical efficiencies and torque ratios. He also compiled a table of torques and mechanical efficiencies for all the epicyclic arrangements of a basic train.

Macmillan also computes the algebraic expressions of efficiencies for all the basic gear train inversions [10]. Similar tables, with an increased number of entries, have been reported by Pollone [13], Müller [14], Maggiore [15], Monastero [16], Pennestrì et al. [4,6], and Esmail [17].

Macmillan observes that the torques acting on the links and power losses are independent of the observer's motion. The following quote is taken from Macmillan [18]

> *...our analysis is based upon an important principle relating to torques and the power lost in friction; this is the fact that magnitudes of the torques acting upon the various members of the gear are quite independent of the motion of the observer who measures them. In addition, the power lost, being determined solely by the internal torques and the relative motions of the wheels within the gear, is also independent of the observer's motion.*

Early applications of kinematic inversion to obtain the mechanical efficiency of epicyclic gear trains are also due to Poppinga [19], Terplan [20], Kudlyavtzev [21], and Looman [22].

More recent applications of the kinematic inversion principle outlined by Macmillan to obtain power losses in epicyclic gearing are due to Yu and Beachley [23] and, Chen and Angeles [24]. In particular, Yu and Beachley [23] introduced the term *latent power* or *gearing power* to denote the power through a link assuming the observer rotating with the gear carrier.