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Research paper

A new analytical model to calculate the maximum tooth root stress and critical section location of spur gear

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ABSTRACT

An accurate calculation of the maximum tooth root stress (TRS) and critical section location (CSL) provides a basis for predicting and improving gear performance. The irregular profile represented by the implicit function may cause the calculation to be more complex. In current research, finite element methods (FEM model) and experimental test methods (ET model) can obtain accurate results but need large computational resources and time. The results from ISO 6336:2006 (ISO model) and AGMA 2101-D04 (AGMA model) are obtained conveniently but sometimes not reliable. Therefore, a new analytical model based on the mechanics theory with an accurate profile equation is established to calculate the maximum TRS and corresponding CSL quickly and accurately by solving the extreme value. Finally, the results of the spur gear in five cases with different parameters are obtained and compared to those of the FEM, ISO and AGMA models. It is shown that the results of the new model are in agreement with those of the FEM model, even under different parametric conditions.

1. Introduction

The TRS is one of the most important gear performance indexes used by gear designers and researchers. A large TRS can directly destroy a gear or indirectly affect gear performance. The accurate and rapid calculation of the TRS and CSL can provide a direct basis for gear structure design. There are currently the following three main calculation models: the FEM model, ET model and analytical model.

The FEM model is commonly used to obtain the TRS and CSL directly and accurately. The influence of different periods, shapes, helical forms and slope deviations on the TRS can be accurately quantified by the FEM model [1,2], which also analyses the TRS of gears that cannot be calculated by other models, such as thin-rimmed spur gears with inclined webs. For this type of gear, the centrifugal load [3] and web angle [4] effects on the TRS at high speeds are measured. Additionally, the FEM model can be applied to reduce the TRS by adjusting the gear parameters to obtain better mechanical properties [5]. The gear tooth profile modification can improve its performance, and the FEM model can evaluate the effect of improvement [6]. In addition, the influence of the tooth profile, machining errors and assembly errors on the TRS are calculated by a three-dimensional FEM model [7]. The FEM model may be able to calculate the load sharing and deformation between teeth

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Nomenclature

AGMA model	Short for 'model based on standard AGMA 2101-D04'
B	Face module [mm]
CSL	Short for 'critical section location'
C_{trc}	Stress concentration factor of the new model
ET model	Short for 'model based on experimental test method'
F	Load in Fig. 4 [N]
FEM model	Short for 'model based on Finite element method'
F_{t1}	The nominal tangential load at the reference cylinder in the ISO model [N]
F_{t2}	Transmitted tangential load in the AGMA model [N]
h_{ga}	Addendum coefficient
HPSTC	Short for 'highest point of the single tooth contact'
ISO model	Short for 'model based on standard ISO 6336:2006'
L_{CSL}	Critical section location of tooth [deg]
L_F	The abscissa of intersection point of the extension line of the load and the X_{gl}-axis in Fig. 4 [mm]
m_n	Normal module [mm]
r_g	Reference circle radius [mm]
r_{ga}	Addendum circle radius [mm]
r_{gb}	Base circle radius [mm]
TRS	Short for 'tooth root stress'
x_g	Addendum modification coefficient
x_{gc}, y_{gc}	The center locations of edge radius of cutter [mm]
x_{ginu}	The parameter equation of the up involute curve on the X-axis
x_{gtru}	The parameter equation of the up transition curve on X-axis
y_{ginu}	The parameter equation of the up involute curve on the Y-axis
y_{gtru}	The parameter equation of the up transition curve on Y-axis
y_{gw}	Half length of the cutter pitch line [mm]
Y_F	Form factor of the ISO model
Y_J	Geometry factor of the AGMA model for bending strength
Y_S	The stress correction factor of the ISO model
z_g	Tooth number
α_g	Normal pressure angle [rad]
α_{ga}	Pressure angle of the addendum circle [rad]
α_{gF}	Pressure angle at the meshing point [rad]
γ_g	Angle of the envelope point [rad]
ρ_{g0}	Edge radius of cutter [mm]
ρ_{tr}	Curvature radius of the down transition curve [mm]
σ_a	Standard axial compressive stress [Mpa]
σ_b	Tooth root stress [Mpa]
σ_{bc}	Total standard tensile stress [Mpa]
$\sigma_{b\text{-}max}$	The maximum tooth root stress [Mpa]
σ_{cc}	Total standard compressive stress [Mpa]
$\sigma_{F0(AGMA)}$	Nominal tooth root stress of the AGMA model [Mpa]
$\sigma_{F0(ISO)}$	Nominal tooth root stress of the ISO model [Mpa]
σ_g	Standard bending moment stress [Mpa]
φ	Independent variable for the curve [rad]
φ_{bgtu}	The value of φ_{gtu} at the CSL of the transition curve [rad]
φ_{gF}	The value of φ_{giu} at the meshing point [rad]
φ_{giu}	Independent variable of the parameter equation of the up involute curve [rad]
φ_{gtu}	Independent variable of the parameter equation of the up transition curve [rad]

for high contact ratio gears that can significantly reduce the TRS, a calculation that is difficult to obtain by other models. Therefore, uniform [8] and profile modifications [9] of high contact ratio gears are included in the scope of research, as well as their optimization for the TRS reduction [10–12]. Finally, in order to lower the computational costs of the FEM model, some new numerical models are also developed [13].

The ET model is another important tool for testing the TRS, and its accuracy is increasingly improved with the development of modern testing technology. The photo elastic method and strain gauging method are widely used; the results obtained by the former directly illustrate the TRS and CSL of a spur gear and are also compared to that of the AGMA model