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Research paper

Geometric isotropy indices for workspace analysis of parallel manipulators

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ABSTRACT

This paper addresses the isotropy of a workspace in terms of its geometric shape. Workspace isotropy analysis plays a crucial role in improving the quality of workspace for parallel manipulators. When a parallel manipulator is installed on a moving platform (e.g., satellites, aircrafts, ships, etc.), the geometry of its workspace is critical to achieve the planned tasks. This paper proposes three novel workspace isotropy indices, namely, translational workspace isotropy index (TWII), rotational workspace isotropy index (RWII), and entire workspace isotropy index (EWII), for workspace isotropy analysis in terms of the geometric shape of the workspace. All three indices are mathematically defined. TWII and RWII can be applied to both planar and spatial parallel manipulators with movable bases while EWII can work for planar parallel manipulators with movable bases. Random rotational disturbances are applied to the movable bases of multiple parallel manipulators in simulation. Simulation results show that the proposed workspace isotropy indices are effective in evaluating how isotropic the geometric shape of a workspace is. They are good at reflecting the robustness of a parallel manipulator to rotational disturbances to its movable base. The proposed indices can also be used as guidelines for the optimal design of parallel manipulators.

1. Introduction

A parallel manipulator has two platforms: the base and the end-effector. Anchor points on both platforms are paired and connected via rigid links for rigid-link parallel manipulators (RLPMs) or cables for cable-driven parallel manipulators (CDPMs) [1,2]. RLPMs have several advantages over serial manipulators, such as high stiffness, large payload, and high accuracy, promoting a wide range of applications [3–6]. RLPMs also have drawbacks such as small workspace, singularities inside the workspace, etc. [3]. Compared with an RLPM, a CDPM usually has larger workspace, lower inertia, larger payload, and more compact actuators. It is also modular, relatively inexpensive, and convenient for re-configuration and transportation [7]. CDPMs are suitable for many applications such as material handling and instrumentation [8–13], but the unidirectional constraint of cables (i.e., cables can pull but cannot push) also introduces many challenges in the design and control of CDPMs.

Technical issues of parallel manipulators, such as kinematics [14], singularity [15], workspace determination [16,17], configuration [18], and trajectory control [19], have been intensively investigated. However, workspace isotropy analysis of par-

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Nomenclature

η_t	Isotropy of the translational workspace on a specific s_r
η_w	Isotropy of the entire workspace on a specific s_r
a_l	Adjusted local rotational workspace
a_r	Space union set of adjusted local rotational workspace on a specific s_r
A_{s_r}	Surface area or circumference of s_r
B_b	Length ratio of the bottom edges to the middle edges of the base polyhedron
b_b	Length of bottom edges of the base polyhedron
B_e	Length ratio of the bottom edges to the middle edges of the end-effector polyhedron
b_e	Length of bottom edges of the end-effector polyhedron
e_b	Length of edges between the vertexes in the middle and the bottom of the base polyhedron
e_e	Length of edges between the vertexes in the middle and the bottom of the end-effector polyhedron
m_b	Length of middle edges of the base polyhedron
m_e	Length of middle edges of the end-effector polyhedron
p_r	Volume of the RWII-analysis space
r	Distance of a position or a pose
r_{max}	Maximum distance of poses in the translational workspace
r_{min}	Minimum distance of poses in the translational workspace
s_r	Spherical surface or a circle on which all positions or poses have the same distance of r
t_d	Translational workspace determination factor
U_b	Length ratio of the top edges to the middle edges of the base polyhedron
u_b	Length of top edges of the base polyhedron
U_e	Length ratio of the top edges to the middle edges of the end-effector polyhedron
u_e	Length of top edges of the end-effector polyhedron
V_{a_l}	Volume of a_l
V_{a_r}	Volume of a_r
w_r	Volume of the rotational workspace

allel manipulators has seldom been touched. This paper deals with workspace isotropy analysis of parallel manipulators, regarding the geometric shape of the workspace.

Workspace isotropy analysis in terms of the geometric shape of the workspace is crucial for many applications in which the base of the parallel manipulator is movable. One of these application scenarios is that the base of a parallel manipulator is mounted on a movable platform, such as a satellite [24], a spacecraft [20], an Unmanned Aerial Vehicle (UAV) [21], and a ship [22], as shown in Figs. 1a–c. In these applications, the base of the manipulator moves together with the movable platform. As a result, the pose (i.e., position and orientation) of the parallel manipulator could be unknown or unpredictable for tasks to be executed due to an unexpected motion of the mobile platform. For example, a gust to the UAV in Fig. 1b may change the attitude of the UAV and make the parallel manipulator unable to reach the pose for aerial manipulation. Thus, it is difficult to determine in advance the ideal poses where the manipulator is convenient to perform these tasks [23]. If the workspace of a parallel manipulator is isotropic in terms of its geometric shape, then unpredictable attitude changes of the movable base of the parallel manipulator due to rotational disturbances applied to the base could be more easily compensated by the parallel manipulator itself.

Another application scenario is when multiple manipulators are connected in serial to conduct complex tasks, such as the cable-driven serial–parallel hybrid manipulators in [24–27]. A sample serial-parallel hybrid manipulator from [26] is shown in Fig. 1d. The isotropy of the workspace of the parallel manipulator in the first level has a decisive influence on the performance of the subsequent manipulators. The parallel manipulator in the first level with an isotropic workspace is a prerequisite for a robust control of subsequent manipulators. Hence, it is important to design the parallel manipulator in the first level such that its workspace is isotropic in terms of the geometric shape of the workspace.

The advantages of a parallel manipulator with isotropic workspace motivate the authors to work on isotropy analysis of parallel manipulators. To better understand and analyze workspace isotropy for parallel manipulators, a clear definition of workspace isotropy is essential. A robotic manipulator is said to be isotropic if its end-effector has the same properties in any arbitrary direction [23]. These properties include, but are not limited to, characteristics of kinematics, dynamics, force, and workspace. This paper proposes workspace isotropy indices to evaluate the workspace of a parallel manipulator in terms of its geometric shape. The workspace of a parallel manipulator is geometrically isotropic about a selected point, called isotropy analysis center (IAC) in this paper, if, with any arbitrary angle of rotation applied to the movable base of the parallel manipulator about the IAC, the workspace after the rotation overlaps the workspace before the rotation. It is assumed in this paper that the base of a parallel manipulator is movable and that the workspace of the parallel manipulator rotates, but does not deform, with the rotation of the movable base. The workspace of a parallel manipulator is assumed to be continuous, rather than a union of a few separated spaces.