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Research paper

Workspace, joint space and singularities of a family of delta-like robot

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A B S T R A C T

This paper presents the workspace, the joint space and the singularities of a family of delta-like parallel robots by using algebraic tools. The different functions of SIROPA library are introduced, which is used to induce an estimation about the complexity in representing the singularities in the workspace and the joint space. A Gröbner based elimination is used to compute the singularities of the manipulator and a Cylindrical Algebraic Decomposition algorithm is used to study the workspace and the joint space. From these algebraic objects, we propose some certified three-dimensional plotting describing the shape of workspace and of the joint space which will help the engineers or researchers to decide the most suited configuration of the manipulator they should use for a given task. Also, the different parameters associated with the complexity of the serial and parallel singularities are tabulated, which further enhance the selection of the different configuration of the manipulator by comparing the complexity of the singularity equations.

1. Introduction

The workspace can be defined as the volume of space or the complete set of poses which the end-effector of the manipulator can reach. Many researchers published several works on the problem of computing these complete sets for robot kinematics. Based on the early studies [1,2], several methods for workspace determination have been proposed, but many of them are applicable only to a particular class of robots. The workspace of parallel robots mainly depends on the actuated joint variables, the range of motion of the joints and the mechanical interferences between the bodies of the mechanism. There are different techniques based on geometric [3,4], discretization [5–7], and algebraic methods [8–12] which can be used to compute the workspace of parallel robot. The main advantage of the geometric approach is that it establishes the nature of the boundary of the workspace [13]. Also, it allows to compute the surface and volume of the workspace while being very efficient in terms of storage space, but when the rotational motion is included, it becomes less efficient.

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Nomenclature

SIROPA	Library for manipulator singularities analysis
CAD	Cylindrical Algebraic Decomposition
IKP	Inverse Kinematics problem
DKP	Direct Kinematics problem
det	Determinant of Jacobian matrix
R	Revolute Joint
P	Prismatic Joint
S	Spherical Joint
ρ	Actuated Joint Variables
X	Pose Variables
A	Direct parallel Jacobian matrices
B	Inverse serial Jacobian matrices

Interval analysis based methods can be used to compute the workspace but the computation time depends on the complexity of the robot and the requested accuracy [7]. Discretization methods are usually less complicated and can easily take into account all kinematic constraints, but they require more space and computation time for higher resolutions. The majority of numerical methods used to determine the workspace of parallel manipulators includes the discretization of the pose parameters for computing workspace boundaries [6]. There are other approaches, such that optimization algorithms [14] for fully serial or parallel manipulators; analytic methods for symmetrical spherical mechanisms [15]. In [16], a method for computing the workspace boundary for manipulators with a general structure is proposed, which uses a branch-and-prune technique to isolate a set of output singularities, and then classifies the points on such set according to whether they correspond to motion impediments in the workspace. A Cylindrical Algebraic Decomposition (CAD) based method is used in [10,17,18] to model the workspace and joint space for the 3-R$\underline{\text{P}}$S parallel robot and delta-like robots. The variations in the workspace, singularities, and joint space with respect to design parameter of a 3-R$\underline{\text{P}}$S parallel manipulator is studied in [19].

Here, this paper presents the results obtained by applying algebraic methods for the workspace and joint space analysis of a family of a delta-like robot including complexity information for representing the singularities in the workspace and the joint space. The CAD algorithm is used to study both the workspace and joint space, and a Gröbner based elimination process is used to compute the parallel and serial singularities of the manipulator. The structure of the paper is as follows. Section 2 presents the mathematical tools and the introduction of SIROPA. Section 3 describes the architecture of the manipulator, including kinematic equation and joint constraints associated with the manipulators. Section 4 discusses the computation of parallel as well as serial singularities and their projections in workspace and joint-space. Sections 5 and 6 present a comparative study on the shape of the workspace and joint space of different delta-like robots, respectively. Section 7 finally concludes the paper.

2. Algebraic tools: SIROPA

SIROPA is a library for the MAPLE developed to analyze the singularities, workspace and joint space of serial and parallel manipulators as well as tensegrity structures [20]. There are two main parts of the library shown in Fig. 1, the first one provides the algebraic tools to solve the constraint equations and convert the trigonometric equations in the algebraic form. The other one, SIROPA, provides modeling, analyzing and plotting functions for different manipulators, shown in Fig. 2. Only a small part of these tools are used in the current paper.

2.1. Modeling functions

SIROPA provides modeling functions such as CreateManipulator(), to virtually create the planar and spatial manipulators for further analysis. Below are the functions:

Functions	
CreateManipulator	Constructs a data structure of type Manipulator
SubsPlus	Substitute coherently angles in a system
UnassignParameters	Specify parameter values in a Manipulator
UnassignParameters	Release parameters in a Manipulator

CreateManipulator

The function **CreateManipulator()** of SIROPA library in MAPLE software is used to virtually create the manipulator for analysis. Listing 1 shows the code architecture of the function.