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Research paper

Influence of joints flexibility on overall stiffness of a 3-PRUP compliant parallel manipulator

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A R T I C L E I N F O

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A B S T R A C T

The main aim of this paper is the influence assessment of the one of the most significant non-geometrical errors called compliance errors on accuracy of a 3-PRUP parallel robot. In this paper, a detailed and comprehensive study is performed to evaluate the effects of the joints and bodies flexibility on position accuracy of the robot. For this purpose, a continuous modeling method based on the energy method and Castigliano's 2nd theorem is presented. The capabilities of this method significantly reduce the limiting assumptions to obtain a highly accurate analytical stiffness model. Using the capabilities of this method, the compliance errors modeling of flexible bodies with complex geometry will be possible as well as the bending and torsional moments, shear forces and weight of all robot compliant modules can be considered as continuous loads. Using the concept of Wrench Compliant Module Jacobian, WCMJ, matrix and physical/structural properties matrix of each module, the compliance matrix of each flexible module is independently obtained. Using a computational algorithm, a comprehensive study is performed to obtain the contribution of the joints flexibility on the robot accuracy throughout the workspace. Finally, to evaluate the compliance errors boundaries throughout the workspace, the concept of General Compliance Error Range, GCER, is presented. The GCER represents a range between maximum and minimum accuracy of robot relative to the compliance errors.

1. Introduction

Generally, the accuracy, repeatability and resolution of a robotic manipulator are of three important criteria to evaluate the manipulators precision. Identification of the effective sources on the robot positioning errors is the first step to improve the robot's accuracy indices. Typically, the dimensional errors, assembling errors and joints clearance are significant types of the geometrical errors as well as the actuators control errors, errors caused by flexibility of robot components, high or low temperatures, external and internal noise in robot control system, measurement errors, errors caused by abrasion, wearing and friction are significant types of the non-geometrical errors [1–4].

In view of the importance and effectiveness of error sources, in robotic machine tools with high rigidity, the compliance errors may be neglected. However, to achieve higher speed and acceleration, weight to stiffness ratio of the robots should be reduced. Hence, in recent years, the applications of parallel robots as CNC machine tools are developed. Although, the stiffness of parallel robots are inherently higher than serial ones, yet, the use of parallel robots as high-speed CNC machine tools require to improve weight to stiffness ratio of these types of robots. When a parallel robot is utilized as high-speed

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Nomenclature

λ_i, φ_i, θ_i	rotation angles of the i^{th} passive R- and U-joints about their z-, y- and x-axes in local coordinate frames $\{T_{1i}\}$, $\{T_{2i}\}$ and $\{T_{3i}\}$.
θ, φ, λ	Euler angles about the x-, y- and z-axes of Moving Star, MS.
${}^{i}_{j}\mathbf{R}$	rotation matrix to transfer a vector defined in $\{j\}$ to $\{i\}$.
U	total strain energy of the manipulator.
U_{bodies}, U_{joints}	strain energy of all compliant bodies and joints of the robot.
U_{MS}, U_{LR}, U_{BS}, U_{AJ}, U_{PJ}	strain energy of MS, LRs, ball screws, active and passive joints.
$\mathbf{w}$	applied external wrench, $\mathbf{w} = \{\mathbf{f}_{ext}^T \quad \mathbf{m}_{ext}^T\}^T$.
$\mathbf{f}_{ext}$, $\mathbf{m}_{ext}$	external force and moment vectors applied to the end-effector.
K, C	overall stiffness matrix and overall compliance matrix of the robot.
$\delta\mathbf{s}$	compliance errors vector of the end-effector due to flexibility of robot's modules.
$\delta\boldsymbol{\chi}, \delta\boldsymbol{\psi}$	translation and rotation compliance errors vectors of the end-effector.
$\delta\mathbf{s}_{bodies}$, $\delta\mathbf{s}_{joints}$	virtual compliance errors vectors due to flexibility of compliant bodies and joints.
$\delta\mathbf{s}_{MS}, \delta\mathbf{s}_{LR}, \delta\mathbf{s}_{BS}, \delta\mathbf{s}_{M}, \delta\mathbf{s}_{PJ}$	virtual compliance errors vectors due to flexibility of MS, LRs, ball screws, motors and passive joints.
$\delta\boldsymbol{\chi}_{bodies}$, $\delta\boldsymbol{\psi}_{bodies}$ & $\delta\boldsymbol{\chi}_{joints}$, $\delta\boldsymbol{\psi}_{joints}$	virtual translation and rotation compliance errors vectors due to flexibility of compliant bodies and joints.
$\delta\boldsymbol{\chi}_{MS}$, $\delta\boldsymbol{\psi}_{MS}$ & $\delta\boldsymbol{\chi}_{LR}$, $\delta\boldsymbol{\psi}_{LR}$ & $\delta\mathbf{s}_{BS}$, $\delta\boldsymbol{\psi}_{BS}$	virtual translation and rotation compliance errors vectors due to flexibility of the MS, LRs and ball screws.
$\delta\boldsymbol{\chi}_{AJ}$, $\delta\boldsymbol{\psi}_{AJ}$ & $\delta\boldsymbol{\chi}_{PJ}$, $\delta\boldsymbol{\psi}_{PJ}$	virtual translation and rotation compliance errors vectors due to flexibility of the active and passive joints.
$\mathbf{C}_{MS}, \mathbf{C}_{LR}, \mathbf{C}_{BS}$, $\mathbf{C}_{AJ}$, $\mathbf{C}_{PJ}$	compliance matrices of MS, LRs, ball screws, active and passive joints.
$\mathbf{f}_{MS}$, $\mathbf{f}_{LR}$, $\mathbf{m}_{LR}$	overall internal reaction forces/torques vectors of MS and LRs.
$\mathbf{f}_{R\theta P}$, $\mathbf{f}_{R\varphi}$, $\mathbf{f}_{R\lambda}$, $\mathbf{m}_{R\lambda}$	overall internal reaction forces/torques vectors of R_θP- and R_φ- and R_λ-joints.
$\mathbf{f}_{BS}$, $\mathbf{f}_{Nut}$, $\mathbf{f}_{SB}$, $\boldsymbol{\tau}_{M}$, $\mathbf{f}_{belt}$	overall internal reaction forces/torques vectors of ball screws, ball nuts, support bearings, resistant torque in motors and tensile forces in the timing belts.
$\mathbf{J}_{MS}^{w}$	Wrench Jacobian matrix of the compliant MS, $WCMJ_{MS}$
$\mathbf{J}_{LR}^{fw}$, $\mathbf{J}_{LR}^{mw}$	Wrench Jacobian matrices of the compliant LRs, $WCMJ_{LR}$
$\mathbf{J}_{R\theta P}^{w}$, $\mathbf{J}_{R\varphi}^{w}$	Wrench Jacobian matrix of compliant R_θP- and R_φ-joints, $WCMJ_{R\theta P}$ and $WCMJ_{R\varphi}$
$\mathbf{J}_{R\lambda}^{fw}$, $\mathbf{J}_{R\lambda}^{mw}$	Wrench Jacobian matrix of compliant R_λ-joints, $WCMJ_{R\lambda}$
$\mathbf{J}_{BS}^{w}$, $\mathbf{J}_{Nut}^{w}$, $\mathbf{J}_{SB}^{w}$, $\mathbf{J}_{M}^{w}$, $\mathbf{J}_{belt}^{w}$	Wrench Jacobian matrix of compliant ball screws, $WCMJ_{BS}$, ball nuts, $WCMJ_{Nut}$, support bearings, $WCMJ_{SB}$, motors, $WCMJ_{M}$ and timing belt, $WCMJ_{belt}$

precise CNC machine tool, stiffness of the robot is considered one of the key design features to improve the accuracy of these robots [5–10]. From the perspective of error compensation, the compliant errors are the compensable errors and the efficient methods such as adjusting the actuators or controller inputs as well as direct modification of geometrical parameters of robot are employed to compensate these errors.

The importance of structural stiffness of the robot on its accuracy has led to widespread researches in this area. In general, structural stiffness modeling methods of robotic systems can be categorized into three approaches called analytical, non-analytical and semi-analytical methods. These methods are categorized as illustrated in Fig. 1.

The analytical methods can be categorized into two approaches such as lumped [11–14] and distributed [5,6,15,16] modeling methods. It is noteworthy that unlike the lumped modeling method, the capabilities of distributed model significantly reduce the limiting assumptions for calculation of the overall stiffness matrix of robotic system. Furthermore, the Finite Element Analysis, FEA, method [17–19] can be referred to as a non-analytical method. In the FEA method, all compliant modules including compliant bodies and compliant joints can be modeled with their true shapes and dimensions. Yet, in spite of the high accuracy of the FEA to obtain the stiffness model of the robot, this method is a time consuming method due to re-meshing the finite element model of robot structure and re-computing for each robot configuration throughout its workspace [10,19,20].

The methods such as Matrix Structural Analysis, MSA, [21–23], Virtual Joints Modeling method, VJM, and combined methods based on FEA [2,24] can be referred to as semi-analytic methods. The MSA method is based on the FEA, with this difference that the MSA method operates with large compliant elements such as beams, cables, *etc.*, to reduce the computational time but the FEA uses small finite elements to design the robot modules [2,25]. This method can lead to obtain an analytical