Mechanism and Machine Theory 126 (2018) 141–154

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Dynamic modeling of planar closed-chain robotic manipulators in flight and impact phases

A.M. Shafei [a,*], H.R. Shafei [b]

[a] Department of Mechanical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran
[b] Department of Mechanical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Hafez Ave, Tehran, Iran

ARTICLE INFO

Article history:
Received 12 October 2017
Revised 18 March 2018
Accepted 20 March 2018

Keywords:
Closed-chain
Impact phase
Flight phase
Gibbs–Appell
Jacobian

ABSTRACT

This paper presents a systematic procedure for the dynamic modeling of a closed-chain robotic system in both the flight and impact phases. In the proposed approach, first, a planar closed-chain system constructed of n rigid links and $n+1$ revolute joints is virtually converted to an open kinematic chain. Then, a recursive algorithm based on the Gibbs–Appell principle is applied to reduce the computational load of the developed model, which contains finite constraints. The normal impact phase of this closed-chain robotic system, which includes impulsive constraints, is modeled based on the Newton's kinematic impact law. Finally, computer simulations of a hexagonal closed-chain robotic system in both the flight and impact phases are carried out. This work is actually an extension of the previous investigation of the authors, which was restricted to the study of an open kinematic chain. So, in order to avoid duplication, only the necessary modifications in converting open kinematic chains to closed-chain robotic manipulators are presented here. To the best of our knowledge, this is the first time that a combination of finite and impulsive constraints in a closed-chain mechanical system has been recursively formulated. Since experimental verification of the proposed model is not easy to achieve in this case and no well-defined benchmark problems could be found to validate the model, the findings of this work are compared with the results obtained by simulating the same system in the "Working Model" computer software.

1. Introduction

Due to their greater agility and accuracy, relative to open-loop manipulators, closed-chain robotic systems are vastly used today in various applications, especially in walking or running mechanisms like the Jansen's "Linkage Walking Robot" [1] or "Sphero Riding Strandbeest" [2]. Moreover, walking or running systems that imitate the type of locomotion in quadruped animals or humans are composed of two distinct flight and impact phases. Although a large number of research studies exist in the field of closed-chain robotic systems, almost none of them have considered the effects of impact on such robotic systems. Indeed, due to the presence of constraints in these robotic systems, their kinematics, dynamics and control are extremely complicated to ascertain. These constraints, which originate from the closed-loop nature of such robotic systems (finite constraints) as well as their collisions with surrounding surfaces (impulsive constraints), make the analysis of these robotic systems more complex, especially when the number of constituent links of these mechanisms increases.

---

[*] Corresponding author.
E-mail address: shafei@uk.ac.ir (A.M. Shafei).

https://doi.org/10.1016/j.mechmachtheory.2018.03.007
0094-114X/

The kinematics, control, and the singularity of closed-chain robotic manipulators with complex configurations have attracted the attention of many researches (see, for instance, [3–12]). However, these topics are not the subject of the present paper, and for more detailed studies one may be referred to the work of Briot and Khalil [13] in which a complete and comprehensive literature review of these subjects has been presented. Compared to the extensive literature on the kinematics of closed-chain robotic systems, relatively few studies exist on the dynamics of such systems. Nevertheless, considering the finite constraints of these robotic systems, their dynamics are still an open subject for research. Most researchers have used well-known dynamic methods and general equations in their works to extract the dynamic models of closed-chain robotic manipulators. These methods and equations include the Lagrange-Euler formulation used by Raoofian et al. [14], Abdellatif and Heimann [15], Li et al. [16] and Ghorbel et al. [17]; the Newton–Euler equations used by Dasgupta and Choudhury [18], Gosselin [19], He et al. [20] and Chaudhary and Saha [21]; the principle of virtual work used by Wang and Gosselin [22], Geike and McPhee [23] and Staicu [24]; and the Kane's equation used by Guang-chun et al. [25], Chang et al. [26] and Anderson [27]. However, despite the great developments achieved by these research studies, most of them suffer from high computational loads, which are due to the combined use of differential and algebraic equations for the dynamic modeling of these kinds of robotic systems; especially when a large number of links has to be simulated.

The dynamic modeling of impact phenomenon, which involves impulsive constraints in multibody systems, has received less attention compared to the modeling of finite constraints. The first attempts for the modeling of impact in multibody systems has been made by Wittenburg [28] who has used the Newton–Euler formulation to derive the motion equations under impact conditions. Haug et al. [29] applied the principles of virtual work for the modeling of impact in multibody systems. By employing the Lagrangian methodology, Rismantab-Sany and Shabana [30] analyzed impulsive motion in nonholonomic mechanical systems. The effect of impact in unconstrained robotic systems has been studied by Chang & Huston [31] via Kane's formulation. Lankarani and Nikravesh [32] applied the Hertz contact law to simulate the motion of multibody systems under impact condition. Recently, Shafei and Shafei [33–35] have employed the Newton's kinematic impact law for the dynamic modeling of impact phenomena in open-chain robotic systems composed of multiple rigid or flexible links. For more studies on this subject, one may refer to the works of Hurmuzlu and Marghitu [36], Rodriguez and Bowling [37], Gloker [38], Goswami et al. [39], Westervelt et al. [40], Gattringer et al. [41] and Tlalolini et al. [42], who have explored the impact phenomenon in multi-rigid-link robotic manipulators. However, the main objective of all the above works has been to improve the modeling of impact in open-loop robotic systems; and closed-chain multibody systems have not been investigated.

Although, the motion equations that are derived through different formulations are equivalent and independent of these formulations, they have different forms; and as a consequence, various algorithms can be used to systematically derive the motion equations. There are numerous recursive formulations that can be applied to closed kinematic chains. Examples of these recursive algorithms can be found in the works of Bhattacharya et al. [43], Ibrahim and Khalil [44], Park et al. [45], Bhalerao et al. [46], Khan et al. [47], Omar [48], Rodriguez et al. [49], Critchley and Anderson [50], Wang et al. [51] and Saha and Schiehlen [52]. However, the emphasis of this paper is on the recursive Gibbs–Appell method, which has been used the least among the other approaches. In deriving the dynamic equations of motion by Gibbs–Appell formulation a Gibbs function (energy of acceleration) is defined first. Next, a set of independent quasi- velocities (linear combination of generalized velocities) is selected. The dynamic motion equations are then obtained by taking the derivative of the Gibbs function with respect to quasi-accelerations (time derivate of quasi-velocities) and equalizing them with generalized forces. In a study related to robotics, Mata et al. [53] presented a recursive algorithm for deriving the inverse and forward dynamic equations of n-rigid-link robotic manipulators by means of the recursive Gibbs–Appell formulation. They showed that their proposed algorithm requires a smaller number of algebraic operations, compared to other methods. This algorithm, which is based on 3×3 rotational matrices, was later extended by Korayem and Shafei [54] to the dynamic modeling of multi-flexible-link robotic manipulators; and its improved computational efficiency relative to Lagrangian formulations was demonstrated. In successive works of the same authors, the said algorithm was effectively applied for the symbolic dynamic modeling of different robotic systems including mobile-base robotic manipulators [55,56], robotic manipulators with revolute-prismatic joints [57,58] and tree-type robotic systems [59]. The main feature of the dynamic algorithms based on 3×3 rotational matrices is their ability to improve the computational efficiency by using less multiplications and additions. However, these kinds of dynamic models suffer from lengthy formulations. So, the abovementioned researchers have also developed an algorithm based on 4×4 transformation matrices and Gibbs–Appell methodology to derive the motion equations in more compact form than 3×3 rotational matrices; albeit at a higher computational cost [60,61]. Nevertheless, all these methods are restricted to open kinematic chains, and they have not been extended to closed-chain robotic manipulators.

The main contribution of this paper is to derive the dynamic models of closed-chain robotic systems in the flight and impact phases in a computationally-efficient manner and without losing generality. Gibbs–Appell formulation is used to recursively and systematically derive the flight phase motion equations in constrained form and the Newton's kinematic impact law is applied to model the impact phase. As was mentioned before, this work is an extension of Shafei and Shafei [33], which was restricted to open kinematic chains. So, instead of writing repetitive formulations, the proposed procedures for the dynamic modeling of closed-chain robotic systems are presented in graphical format. Finally, a closed-chain robotic system composed of six rigid links is simulated to verify the proposed method.