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Research paper

Developable compliant-aided rolling-contact mechanisms

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ABSTRACT

Rolling-contact mechanisms can provide low-friction motion with unique kinematic paths. We show that developable surfaces can be used as a design tool for rolling-contact mechanisms joined with compliant bands. These mechanisms can exhibit 3D motion paths, couple rotational and translational motions into a single degree of freedom, and can be designed to exhibit various tailored kinetic responses. We set forth developable surface parametrizations well suited to the creation of rolling contacts. We highlight how the geodesic and principal curvatures of the non-ruling principal curves of a developable surface are meaningful design quantities for rolling contacts. We provide kinematic and kinetic analyses and demonstrate several developable compliant-aided rolling-contact mechanisms in physical prototypes.

1. Introduction

Rolling-contact mechanisms are characterized by surfaces exhibiting pure rolling relative to each other to create relative motion between bodies. These types of mechanisms have proven useful in applications where unique motion paths are required, friction needs to be kept to a minimum, and under high compressive loads. For example, an artificial disc replacement rolling mechanism was created for degenerated vertebral discs that mimics the motion of a healthy disc [1]. Compliant-aided rolling-contact mechanisms use flexible bands or segments to help enforce the rolling constraint. Existing compliant-aided rolling-contact mechanisms commonly utilize cylindrical rolling surfaces to achieve motion [2–4]. Recently rolling-contact joints created from generalized cylindrical surfaces, surfaces formed from translating a generator line such that all ruling lines on the surface are parallel to each other, were employed to create the required panel offsets for a thickness accommodation technique for origami vertices [5]. In addition Lang et al. set forth theorems for 3D rolling motion [5]. This paper presents a method that illustrates how developable surfaces can be used to design a family of compliant-aided rolling-contact mechanisms with both planar and 3D motion. Specific kinetic responses can also be designed by modifying the stiffness of the flexible bands and curvature of the rolling surfaces [6].

In Section 2 we give a brief background of rolling mechanisms and developable surfaces. In Section 3 we set forth a mathematical notation for developable surfaces convenient for rolling contacts, discuss how rolling-contact mechanisms can be created using developable surfaces as a basis, give kinematic and kinetic models, and discuss a special case force response, static balance. In Section 4 we show several rolling-contact mechanisms and discuss their construction. Lastly we close with a final discussion and mention areas of potential future work.

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2. Background

2.1. Rolling-contact mechanisms

Mechanisms incorporating rolling into their motion can provide unique benefits for engineering applications. For example, ball and other rolling bearings have been commonly used to reduce friction during movement in the past [7] and continue to be researched [8]. Cams and followers provide specific cyclic motion and have been highly utilized in engines [9]. Cylinders rolling on each other, implemented in forms such as gears, can create dramatic changes in mechanical advantage and change directions of motion [10].

One class of rolling-contact mechanisms uses compliant flexures to enforce a rolling constraint between two surfaces. For example, a Jacob's Ladder toy can be considered a compliant-aided rolling-contact mechanism [11]. The compliant rolling-contact element (CORE) [12] and X_r joint [3] use the same type of flexure architecture as the Jacob's Ladder toy to create a joint with two rolling-contact surfaces which curve away from each other, such as two circles. Multiple stable locations can be built into these compliant-aided rolling-contact joints [4]. Another flexure arrangement is seen in the Rolamite joint, which exhibits linear motion with low friction [6]. Advantages of elliptical rolling-contact joints over circular joints are explored by Montierth et al. [13]. An interesting analysis of planar rolling-contact mechanisms has been written by Kuntz [14].

It should be noted that compared to a traditional revolute joint where the axis of rotation is stationary throughout the motion, rolling-contact mechanisms have an instantaneous axis of rotation which continuously moves locations as the surfaces roll across each other [15]. This quality gives rise to unique motion paths, but also can complicate actuation and implementation.

In addition, rolling-contact mechanisms have the ability to take complex motions which are normally two degrees of freedom (translation and rotation), and reduce the system to a single degree of freedom, provided that a no-slip condition is enforced during the rolling motion [14].

Compliant-aided rolling-contact mechanisms have been used in multiple applications. Many of these applications involve some degree of biomimicry, as rolling motions are readily seen in biological joints. A prosthetic knee joint was designed by Hillberry and Hall [16]. More recently rolling joints were used to mimic the motion of a knee in the creation of a knee prosthesis and brace [17–19]. Finger joints have been constructed out of cylindrical rolling elements joined by flexible bands [2,20]. Rolling joints were employed in the creation of a human-oriented biped robot [21]. Steerable surgical tools have utilized rolling architectures to facilitate small part sizes [22,23]. A grapser with force perception was designed using compliant-aided rolling contacts [24]. A deployable compliant-aided rolling-contact joint was created by employing curved-folding origami techniques [25]. Folding plate structures have been facilitated using rolling-contact mechanisms [26]. Pellegrino et al. created a compliant-aided rolling-contact joint suited to deploying structures in space using a tape spring for actuation [27].

Recent work on a thickness accommodation technique for origami patterns used compliant-aided rolling-contact mechanisms to create offsets between panels as they moved into a stacked position [5]. As part of this work two rolling-contact theorems were set forth. The first describes how relative planar motion between two bodies that is well-behaved, meaning the motion never is pure translation, but is always coupled with a rotational component, can be accomplished through two translationally symmetric surfaces rolling on each other. These translationally symmetric surfaces are ruled surfaces and can furthermore be described as generalized cylinders. The second theorem states that relative 3D motion between two bodies that is well behaved and satisfies a no-lateral sliding condition can be created through rolling two ruled surfaces upon each other. Besides stating these two general theorems, their application was shown in the paper through the design of rolling surfaces which match the kinematics of origami vertices.

2.2. Developable surfaces

Rather than creating specific rolling-contact pairs which satisfy origami-vertex kinematics, in this paper we will discuss how developable surfaces can be used to create a family of compliant-aided rolling-contact pairs. A developable surface is a surface that can be created by bending a plane without any stretching or tearing [28]. Developable surfaces are composed of ruling lines (straight lines) making them also ruled surfaces [29]. Planes, generalized cylinders, generalized cones, and tangent developables are the four basic classes of developable surfaces [30]. The three classes other than planes are shown in Fig. 1. General or composite developable surfaces can be created by splicing these classes together using ruling lines or curved creases [31,32].

The traditional parametric representation of developable surfaces takes the form of

$$\mathbf{r}(t,u)=\boldsymbol{\alpha}(t)+u\boldsymbol{\beta}(t) \tag{1}$$

where t and u are the parameters of the surface, $\boldsymbol{\alpha}(t)$ is the directrix of the surface, and $\boldsymbol{\beta}(t)$ is the direction of the ruling lines.

Its often convenient to represent developable surfaces with their *canonical parametrization* $\mathbf{r}(t,v)=\hat{\boldsymbol{\alpha}}(t)+v\hat{\boldsymbol{\beta}}(t)$ [33]. In this form we require $\hat{\boldsymbol{\beta}}(t)$ be a unit vector and change u to v to reflect that v is now an arc length along a ruling line. Additionally for generalized cylinders $\hat{\boldsymbol{\alpha}}(t)$ (representing a cross section of the cylinder) is orthogonal to $\hat{\boldsymbol{\beta}}(t)$, a constant.