Mechanism and Machine Theory 126 (2018) 273–294

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Introducing mass parameters to Pseudo–Rigid–Body models for precisely predicting dynamics of compliant mechanisms☆

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ARTICLE INFO

Article history:
Received 1 January 2018
Revised 22 March 2018
Accepted 6 April 2018

Keywords:
Compliant mechanisms
Parallel-guiding mechanisms
Pseudo–Rigid–Body model
Mass property parameters
Continuum model
Dynamics
Natural frequency

ABSTRACT

In this paper, we introduce non-dimensional mass property parameters to the classical PRB model for accurately predicting the dynamics of the compliant mechanisms. Given extensive work on PRB models accurately modeling the statics and kinematics of compliant mechanisms, few works have investigated on their accuracy in predicting dynamics. Here we study the fundamental natural frequency of two typical mechanisms: fixed-root and pinned-root compliant parallel-guiding mechanisms (CPGM). First, analytical expressions of the natural frequencies of the continuum models are derived according to the vibration mechanics. Second, theoretical expressions of the natural frequencies of the parameterized PRB model are obtained from their dynamics equations. Third, the mass property parameters are optimized to minimize the error between the continuum models and the parameterized PRB model. We conclude that the optimized PRB models can well predict the dynamics especially the low input signal frequency or mass of beams are not negligible. These new PRB models can significantly improve computational efficiency in dynamics simulation of compliant mechanisms.

1. Introduction

Compliant mechanisms have numerous applications in mechanical designs [1–3] and robotic systems [4–6] due to their strengths including reduced part count, less friction, and easy for assembly with high precision and reliability [7]. However, compliant mechanisms are relatively difficult in analyzing, designing, and modeling, especially the dynamic modeling due to the infinite degree of freedom (DOF). To address kinetostatic analysis of the compliant mechanisms, finite element analysis (FEA) [8] method and the Pseudo–Rigid–Body (PRB) model [9,10] are two commonly used approaches. In addition, the Chained–Beam–Constraint–Model (CBCM) is recently developed [11,12] for the static analysis of compliant beams. However the FEA method requires significant computational load to obtain an acceptable accuracy. The PRB model method, on the other hand, has been developed and proven as an efficient approach for design and kinetostatic analysis of the compliant mechanisms with a moderate accuracy.

Recent developments of PRB models on statics and kinematics include the 2R PRB model [13], the 3R PRB model [14], the RPR model considering beam extension or beam circular shape [15,16], etc. However, research on the dynamics of the

☆ This work was conducted in The Ohio State University.

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https://doi.org/10.1016/j.mechmachtheory.2018.04.005

Nomenclature

A	cross section area of the beam
E	Young's modulus of the material
I	second moment of inertia
$\boldsymbol{K}$	stiffness matrix of the dynamics equation
L	length of the beam
$\boldsymbol{M}$	inertia matrix of the dynamics equation
a	location of the mass center
h	height
m	mass
t	thickness
w	transverse displacement
y	the mode wave number
α	ratio of distance of the mass center to the length
β	ratio of the mass of the PRB segment to the beam mass
γ	characteristic radius factor
δ	the mode frequency error between the continuum model and the simulation model
λ	ratio of the tip mass to the beam mass
ρ	density of the material

Subscripts

b	beam
i	counters
t	beam tip

Superscripts

p	the PRB model
c	the continuum model
s	the simulation model

compliant mechanisms using PRB models is limited and particularly on the accuracy study [17]. Lobontiu [18] studied the dynamics of flexure hinges including kinetic energy, free/forced response, and damping effects. Li and Kota [19] investigated the natural frequency characteristic and the dynamic response of the compliant mechanisms using the FEA method. Rösner et al. [20] derived a dynamic model of flexure hinges via the FEA method. Zhao et al. [21] developed the dynamic model of a compliant linear-motion mechanism using the Lagrange equation. In addition to the traditional approaches on dynamic analysis, a few studies have investigated the PRB model on dynamic analysis of compliant mechanisms. Boyle et al. [22] presented the dynamics of a constant-force compliant mechanism with a generalized PRB model, but effects of the input signal's parameters and the design parameters are relatively less explored. Yu et al. [23] studied the dynamics of the PRB model of the compliant mechanisms, and Lyon et al. [24,25] studied the first modal frequency of compliant mechanisms based on the PRB model. However, both of them only studied the natural frequencies and no dynamic performance and accuracy were investigated. She et al. [26] studied the dynamics of the 3R PRB model, but the accuracy of dynamics of the PRB model only held for a short time period and no explorations were conducted on the effects of the design or input parameters on the output performance or accuracy. She et al. [27] first introduced mass property parameters to the original PRB model, but the work was limited to the optimization of the fundamental natural frequency, and no dynamic performance was explored. Li et al. [28] investigated the dynamics of the PRB model considering the design parameters such as the mass ratio of the beam mass over the end mass. However, they did not derive the analytical expression of the natural frequency in terms of design parameters.

In this paper, we present a systematical procedure for developing optimized PRB models that can precisely predict the dynamics of compliant mechanisms. We introduce a set of mass property parameters to the classical PRB model such that the fundamental natural frequency of the optimized PRB model agrees with that of the continuum model. The organization of the paper is described as the following. We begin with the problem statement in Section 2. Theoretical expressions of the continuum model with its natural frequency are derived in Section 3. Parametrized PRB models with analytical expressions of the natural frequency of the compliant beams are presented in Section 4. Dynamics validations and evaluations are conducted in Section 5. Finally, conclusions and future work are presented in Section 6.

2. The problem statement

It is well known that the conventional PRB models predict the static force deflection of a flexible continuum member using a serial chain of two or more rigid links joined by torsion/extension springs [29]. In these PRB models, two sets