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Research paper

Resolution of redundancy in robots and in a human arm

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ABSTRACT

The author obtained his Ph. D. in 1986 under Professor Bernard Roth. In a publication from the research, it was shown that the redundant joints in a serial robot could be used to make the end-effector linear velocity distribution isotropic. In this work we revisit those results, present recent results on how redundancy is made use of in a human arm, and finally attempt to link the earlier work with new results. The human arm can be modeled as a redundant serial manipulator and the redundancy can be computed from the null-space of the Jacobian matrix. In a recent work, healthy adults were made to perform point-to-point reaching tasks in eight directions first without any disturbance, then with an applied force and finally with the force switched off. Statistical analyses show that trajectory and reaching errors due to the applied force die out with trials and subjects who explore the redundancy in the arm adapt faster to the external force. It is also shown that the anisotropy in the error distribution reduces with trials. These new results suggest that the redundancy in a human arm is used to reduce trajectory errors and anisotropy arising out of external disturbances.

1. Introduction

To position and orient a rigid body in three dimensional space, one needs to specify six parameters – three for position and three for orientation. A consequence of this fact is that a six degree-of-freedom robot, with six independently actuated joints, can arbitrarily position and orient an end-effector or an object carried by the end-effector in three-dimensional space. However from early days, robots with more than six actuated joints have been built and in biological systems, more than the required number of actuated joints is ubiquitous. Such systems are known as redundant systems and a key problem in redundant systems is that for a specified position and orientation of the end-effector, there exists infinitely many possibilities for the actuated joints. To choose one particular set, also known as *resolution* of redundancy, researchers have proposed many strategies and extensive research continues to be done. One of the earliest proposed use of redundancy was to overcome the constraints on motion of the robot end-effector due to the presence of joint limits [1] and to avoid obstacles and singularities present in the workspace of a robot [2]. Some of the strategies, such as obstacle avoidance and avoiding wrist singularities were implemented on a seven degree-of-freedom prototype robot [3] and on a prototype wrist with four joints [4]. The key mathematical tool used by most of the researchers in the 80's was the Moore–Penrose generalized inverse [5], also called the *pseudo-inverse*, of the manipulator Jacobian matrix. The manipulator Jacobian matrix relates the end-effector linear and angular velocities to the joint rates and for a redundant robot, the manipulator Jacobian matrix is *rectangular*. Hence, for a prescribed end-effector linear and angular velocity, the Jacobian matrix cannot be inverted to obtain the joint rates. In its basic form, the use of the pseudo-inverse of the Jacobian matrix can be shown to minimize

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the sum of squares of the joint rates for a given linear and angular velocity of the end-effector. The pseudo-inverse of the manipulator Jacobian matrix were also used to minimize joint acceleration, used with weighting matrices to minimize joint torques and extended to include a null-space term which could be used to optimize additional quantities such as a manipulability index (see the review paper by Klein and Huang [6] and textbook by Nakamura [7] and the reference contained in them for details of various pseudo-inverse based resolution schemes in redundant robots). The pseudo-inverse based approaches are numerical in nature and provide results at the level of joint velocities – they do not provide any insight or give deeper understanding at the level of position or orientation of the end-effector. The pseudo-inverse involves obtaining the inverse of a matrix and this has a complexity of $\mathcal{O}(n^4)$ where n is the number of joint variables. Hence it is not very efficient for modeling and simulation of hyper-redundant systems which have a much larger number of links and actuated joints – examples of a hyper-redundant systems are "snake" robots [8], models of continuum robots [9], and classical models of proteins [10] with chains of amino acids during folding.

To overcome some of the difficulties in the pseudo-inverse based approach, researchers proposed an approach where the central 'backbone' of a hyper-redundant manipulator is approximated with a continuous curve. The redundancy is resolved by updating the curve for a desired motion of the end-effector and at every step *fitting* a robot with rigid links and joints. In reference [11], the backbone curves was chosen as linear combination of modes and as splines in reference [12]. In this approach, since the continuous curve is used for motion planning the axial *length* of the curve and as a consequence the length of the hyper-redundant manipulator is not preserved.

A third resolution scheme for hype-redundant manipulators, based on the classical *tractrix* curve, was proposed in reference [13]. Unlike the joint space pseudo-inverse based schemes, this is a Cartesian space scheme. For a prescribed Cartesian motion of the tip of the first link of the hyper-redundant robot (termed as the 'head'), the Cartesian motion of the end of the first link (termed the 'tail') is computed according to the closed-form equations of the tractrix curve. The motion of the 'head' of the second link is then set to the motion of the 'tail' of the first link and again the motion of the 'tail' of the second link is computed according to the equations of the tractrix curve. Proceeding in a similar way, the Cartesian motion of all the links of the hyper-redundant robot is obtained. Once the Cartesian motion is obtained, the rotations at the joints can be obtained from simple trigonometry and vector algebra. It is shown in reference [13] that the tractrix based approach has a complexity of $\mathcal{O}(n)$ and, more interestingly, the motion along the chain dies out as one traverses the chain from the end-effector to the other end. This property makes the motion of the entire hyper-redundant manipulator more natural. In a series of papers, the author and his students have used the tractrix based approach to perform real-time and realistic simulations of one-dimensional flexible objects such as ropes and hyper-redundant manipulators [14], compared the pseudo-inverse, modal and tractrix based approaches on a 8 link planar hyper-redundant robot [15], shown that the tractrix solution results from a general variational problem where a functional defining the infinitesimal motion of the points on a continuous curve is minimized subject to preservation of the arc length of the curve (for a straight line segment, the velocity of the 'tail' lies along the straight line segment [16]), used splines and the tractrix based approach to obtain efficient algorithms for simulation and rendering of the motion of flexible one-dimensional objects [17] and, finally, shown that the tractrix based approach can be extend to include obstacle avoidance [18].

The linear and angular velocity distribution of an end-effector in a serial manipulator is governed by the nonlinear kinematic equations. It has been known that the velocity distribution is not uniform and some directions are easier to move than others [19]. In reference [20], the authors had proposed a redundancy resolution scheme where a redundant joint rate was chosen in a particular way to make the end-effector velocity distribution *isotropic*. In this work, we revisit this approach and bring out its key features – this is the content of Section 2. In Section 3, an experiment on point-to-point reaching along randomly chosen eight directions in a plane, conducted by healthy human subjects, is presented. A model of the redundant human arm is considered and an approach to quantify the redundancy present in the human arm model, using the null-space of the Jacobian matrix, is proposed. When a lateral disturbing force is applied during the reaching tasks, the error in the trajectory initially increases and as the trials progress, the error reduces. In Section 3, from statistical analyses, it is shown that subjects who explore redundancy during the un-perturbed motion, learn how to reduce the error arising out of the disturbance faster. It is also shown that errors in certain directions are consistently larger and the anisotropy in the error distribution decreases with the trials. Taken together these new results suggest that redundancy in human arm is used for learning how to deal with perturbations and one possible mechanism of reducing trajectory errors is by reducing the anisotropy in error inherently present in the point-to-point reaching tasks. Section 4 presents the conclusion of this work.

2. Resolution of redundancy and isotropy

In this section, we present the notion of velocity distribution of the end-effector of a serial robot and the approach to use the redundancy to make the velocity distribution isotropic. More details are available in references [19,20].

2.1. Velocity ellipse

Consider a simple planar two-degree-of-freedom manipulator with two rotary joints as shown in Fig. 1. The end-effector point (x, y) can be written in terms of link lengths l_1, l_2 and the joint angles θ_1 and θ_2 as

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$

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