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Research paper

Kinetostatic modeling of complex compliant mechanisms with serial-parallel substructures: A semi-analytical matrix displacement method

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ABSTRACT

Kinetostatic analysis of compliant mechanisms are crucial at the early stage of design, and it can be difficult and laborsome for complex configurations with distributed compliance. In this paper, a kinetostatic modeling method for flexure-hinge-based compliant mechanisms with hybrid serial-parallel substructures is presented to provide accurate and concise solutions by combining the matrix displacement method with the transfer matrix method. The transition between the elemental stiffness matrix and the transfer matrix of flexure hinges/flexible beams is straightforward, enabling the condensation of a hybrid serial-parallel substructure into one equivalent two-node element simple. A general kinetostatic model of the whole compliant mechanisms is first established based on the equilibrium equation of the nodal force. Then, a condensed two-port mechanical network representing the input/output force-displacement relations of single-degree-of-freedom (DOF) compliant mechanisms and the Jacobian matrix for multi-DOF compliant mechanisms are respectively built. Comparison of the proposed method with the compliance matrix method in previous literature, finite element analysis and experiment for three exemplary mechanisms reveals good prediction accuracy, suggesting its feasibility for fast performance evaluation and parameter optimization at the initial stage of design.

1. Introduction

Unlike traditional rigid-body mechanisms, compliant mechanisms transmit motion, force and energy through the elastic deformation of their components. Compliant mechanisms can provide highly accurate smooth motions without wear, friction, backlash and often need no assembly [1]. Therefore, they have attracted widespread attentions in variety of scientific and industrial applications, including precision positioning stage, micro griping manipulation, precision manufacturing [2–5], and so forth. However, one of the disadvantages in comparison to conventional rigid-body mechanisms is that their design and analysis require simultaneous consideration of kinematic and elasto-mechanical behaviors.

Compliant mechanisms are often configured serially and/or in parallel with various kinds of flexure hinges. The kinetostatics of these flexure-hinge-based compliant mechanisms can be analyzed similar to the multi-rigid-body mechanisms by

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modeling the flexure hinge as an equivalent joint with springs based on the pseudo-rigid-body model (PRBM) [1]. Serial compliant mechanisms can also be easily analyzed by employing the transfer matrix method or the chain algorithm [6]. However, for complex configurations with serial-parallel substructures and distributed/hybrid compliance, which are widely applied in engineering, the above mentioned modeling methods are incomplete and other methods are needed.

Finite elemental analysis is one of the most popular methods for analyzing compliant mechanisms. It can provide excellent solutions for general structures with no constraint on geometry. However, the procedure is time consuming for frequent iterations at the early stage of design where many concepts should be evaluated in a short period of time. Meanwhile, a sufficiently large number of elements are inevitable for flexure hinges to obtain a reliable result. Thus, it requires too much time to be feasible for fast performance prediction and real-time controller design, though it is often used to verify the performance before fabrication.

Considering the limitations of FEM, theoretical modeling for compliant mechanisms has been an interesting and popular topic since the pioneer works of analytical modeling for flexure hinges by Paros and Weisbord [7] and the subsequent works by Howell, Lobontiu and other scholars [1,8]. Many theoretical methods are now available such as the PRBM [5], the principle of virtual work [9–11], the Castigliano's second theorem [12,13], the compliance matrix method [14,15] and the constraint-force-based method [16]. PRBM was proposed to solve the problem of large deformations [1,17–19] and now it is also used for theoretical modeling of compliant mechanisms by assuming the flexure hinge as a joint with springs [20]. Ma [21], Xu [22], Mottard [23] and Qi et al. [24] established kinetostatic models for rhombic and bridge-type amplifiers based on the elastic beam theory; and the accuracy of these models was enhanced recently by Ling et al. [9]. Meanwhile, the virtual work principle was also used to study the displacement attenuation in multistage compliant mechanisms [25]. Lobontiu and other scholars applied the Castigliano's second theorem to analyze the static performance of typical compliant mechanisms [8,12,13]. Generally, the Castigliano's second theorem and the elastic beam theory are popular for simple structures. For complex compliant mechanisms or their composed systems with all kinds of flexure hinges, kinetostatic modeling can resort to the compliance matrix method. Li and others carried out plenty of kinetostatic modeling for precision positioning stages using the compliance matrix method [14,15,26,27]. Pham et al. [28] and recently Lobontiu [29], Jiang et al. [30] separately proposed kinetostatic modeling methods for complex compliant mechanisms with serial-parallel substructures based on the compliance matrix method. However, the modeling procedures are still complicated. A modeling method similar to the rigid multi-body dynamics proposed by Ryu et al. [31] has been further developed and is now used for static and dynamic analyses of precision positioning stages [32,33]. It should also be noted that only the compliance of the flexure hinge was considered while that of the flexible beam was neglected in most of the aforementioned modeling methods, which may lead to low prediction accuracy.

The contribution of this paper is to present a general kinetostatic modeling method for flexure-hinge-based compliant mechanisms with complex serial-parallel substructures having distributed/hybrid compliance by combining the matrix displacement method with the transfer matrix method, which is different from the compliance matrix method in [14,15,26–30]. The paper is limited to planar mechanisms due to their extensive applications. It can be extended to spatial cases with a similar procedure. Besides, this paper mainly deals with the problem of small deformations. For nonlinearly large deformation, the readers are recommended to refer to the previous works of Howell [1], Awta [34], Chen [35], Su [18] or others [36]. Interestingly, these previous pseudo-rigid-body or nonlinear models would be further modified and included as an element or a module into the presented general model for large deformations.

The paper is organized as follows. A general kinetostatic modeling approach without condensation is presented in Section 2. The condensed modeling procedure is conducted in Section 3. The consideration of the rotational motion for multi-DOF compliant mechanisms is discussed in Section 4. Then, summary of the proposed modeling methodology as well as its numerical and experimental verifications are illustrated in Section 5 and Section 6, respectively. The conclusions are made in the final section.

2. Kinetostatic model based on the matrix displacement method

In applications, most flexure-hinge-based compliant mechanisms are organized serially and/or in parallel. Fig. 1(a) provides the configuration of a piezo-actuated precision positioning stage. It is formed by serial and parallel branch chains with three parts: (i) flexure hinge; (ii) flexible beam (the guiding beam is also termed ‘flexible beam’ in the following); and (iii) lumped mass (e.g. output port). It can represent many cases in applications. Fig. 1(b) is a generic topology abstracted from Fig. 1(a), which will be used as the exemplary configuration for the proposed modeling procedure.

Fig. 2 illustrates the serial and parallel branch chains cut from the topology in Fig. 1(b). A serial branch chain is composed of several flexible beams interconnected by flexure hinges with the characteristics of one junction node connecting only two elements, while one junction node has at least three elements interconnected in a parallel branch chain.

The proposed modeling procedure consists of three steps:

Step 1: Discretization and numbering

A compliant mechanism is first discretized into the flexure hinge, the flexible beam and the lumped mass according to the configuration. The flexure hinge bears the largest deformation and the flexible beam bears different levels of deformation in different configurations. For example, the guiding beams in Fig. 1(a) bear large deformation while other flexible beams have small deformation. In the proposed modeling method, compliances of the flexure hinge and the flexible beam are both considered. The lumped mass bears little deformation compared to the other two elements. As shown in Fig. 3, the flexure

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