Mechanism and Machine Theory 123 (2018) 124–136

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

An approach for elastodynamic modeling of hybrid robots based on substructure synthesis technique

Long Wu [a], Guofeng Wang [a,*], Haitao Liu [a], Tian Huang [a,b]

[a] Key Lab. of Mechanisms Theory and Equipment Design of the State Education Ministry, Tianjin University, Tianjin 300072, China
[b] School of Engineering, The University of Warwick, Coventry CV4 7AL, UK

ARTICLE INFO

Article history:
Received 21 September 2017
Revised 12 December 2017
Accepted 20 December 2017

Key words:
Substructure synthesis
Modal reduction
Virtual joints
Parallel kinematic machines

ABSTRACT

Parallel kinematic machines exhibit strong pose-dependent static and dynamic behaviors, which makes accurate and rapid compliance analysis over the entire workspace an important issue in the design optimization. This paper presents a general approach for elastodynamic modeling of parallel kinematic machines using substructure synthesis technique. Firstly, the whole system is decomposed into two groups of substructures, i.e. component substructures and joint substructures. Then, the degrees of freedom of component substructures are reduced using modal reduction technique, and joint substructures are modeled by virtual springs with contact stiffness between adjacent component substructures. Finally, two threads are merged at junction surfaces to offer the equations of motion of the system as a whole, allowing the static and dynamic performances to be rapidly predicted with sufficient accuracy. The dynamic model of a 5-DOF hybrid robot is developed using the proposed method and the computational results show that rigidity and lower mode natural frequencies over the workspace match well with those obtained by the full finite element analysis.

1. Introduction

Static and dynamic compliances are two important performance factors of parallel kinematic machines (PKMs) [1–5] especially for those dedicated to metal cutting, high-speed milling or drilling for example, where high rigidity and high dynamics are crucially required. Since static and dynamic behaviors are highly pose-dependent within the work envelop [6–11], the development of effective techniques for dynamic modeling is of great importance in the design optimization as well as in the cutting stability prediction. Literature reveals intensive investigations in the past decades towards dynamic modeling of PKMs. The approaches available at hand can roughly be classified into four categories, i.e., lumped parameter method, semi-analytical method, finite element (FE) method, and substructure synthesis method.

Finite element analysis (FEA) [12–14] is the most accurate method because 3D geometry of links, contact rigidity of joints, and distributed external forces (gravity for example) can be precisely modeled. However, the FE model has to be re-meshed over and over again at different configurations, involving a time consuming procedure. In order to overcome this problem, Ma et al. [15] presented a feature-based CAD-CAE integration method that allows the FE model at different configurations to be updated automatically in a batch manner. The FE model at a given configuration, however, would still have over hundred

* Corresponding author.
E-mail address: gfwangmail@tju.edu.cn (G. Wang).

https://doi.org/10.1016/j.mechmachtheory.2017.12.019

Nomenclature

R	Global reference frame
R^C, R^J_m	Body-fixed frame of component substructure and joint element m.
$\boldsymbol{m}$, $\boldsymbol{k}$	Mass and stiffness matrices of a component substructure evaluated in R^C
$\boldsymbol{u}$, $\boldsymbol{f}$	Nodal displacement and force vector of a component substructure evaluated in R^C
$\boldsymbol{u}_j$, $\boldsymbol{u}_i$	Nodal displacement vector of junction nodes and interior nodes evaluated in R^C
$\boldsymbol{m}_R$, $\boldsymbol{k}_R$	Reduced mass and stiffness matrices of a component substructure evaluated in R^C
$\boldsymbol{u}_R$, $\boldsymbol{f}_R$	Reduced nodal displacement and force vector of a component substructure evaluated in R^C
$\boldsymbol{T}_S$	Equivalent static transformation matrix
$\boldsymbol{\Phi}$, $\boldsymbol{I}$	Mode shape matrix and unit matrix
$\boldsymbol{M}^C$, $\boldsymbol{K}^C$	Reduced mass and stiffness matrices of a component substructure evaluated in R
$\boldsymbol{U}^C$, $\boldsymbol{F}^C$	Reduced nodal displacement and force vector of a component substructure evaluated in R
$\boldsymbol{R}_C$	Orientation matrix of R^C with respect to R
$\boldsymbol{T}_C$	Coordinate transformation matrix of R^C with respect to R
$\boldsymbol{k}_m$	Stiffness matrix of joint element m evaluated in R^J_m
$\boldsymbol{R}_{J,\,m}$	Orientation matrix of R^J_m with respect to R
$\boldsymbol{T}_{J,\,m}$	Adjoint transformation matrix of R^J_m with respect to R
$\boldsymbol{k}^J$	Stiffness matrix of serially connected joint elements evaluated in R
$\boldsymbol{M}^J$, $\boldsymbol{K}^J$	Mass and stiffness matrices of a joint substructure evaluated in R
$\boldsymbol{U}^J$, $\boldsymbol{F}^J$	Nodal displacement and force vector of a joint substructure evaluated in R
$\boldsymbol{U}^V_i$	Nodal displacement vector of virtual node on component substructure C_i
$\boldsymbol{U}^C_{i,j}$	Junction displacement nodal vector of component substructure C_i
$\boldsymbol{U}^C_{i,p}$	Internal elastic modal coordinate vector of component substructure C_i
$\boldsymbol{\varepsilon}_i$, $\boldsymbol{\xi}_i$	Translational and angular nodal displacement vector of virtual node on component substructure C_i
$\boldsymbol{r}_{i,\,q}$	Position vector pointing from the virtual node to the corresponding junction node q
$w_{i,\,q}$	Weight factor for junction node q on component substructure C_i
$\boldsymbol{\lambda}$	Lagrange multipliers
$\boldsymbol{C}$	Displacement operator for substructures

thousand degrees of freedom, causing a high computational cost in static and dynamic analyses. Therefore, the FE model is more suitable for the final design or validation of other static and dynamic analysis approaches.

In order to improve computational efficiency for static and dynamic behavior prediction in the stage of conceptual design, the lumped spring-mass model could be employed for the systems where the dynamic behaviors are presumably dominated by inertia of the movable components and rigidities of joints. The typical PKM that can be modeled as a lumped spring-mass system would be the Stewart platform. Compared with the lumped parameter method, the semi-analytical methods typically represented by Kineto-Elastodynamics (KED) [16–21] and Flexible Multibody Dynamics (FMD) [22,23] are more suitable for formulating dynamic models of planar and spatial PKMs where the component compliances are no longer negligible. In these methods, the movable links are usually modeled by planar and/or spatial beam elements or finite segments, resulting in either the linear or nonlinear equations of motion, depending upon whether or not taking into the coupling effects of rigid motions over the flexible motions. Generally speaking, the FMD method is more general and effective than the KED method because the constraints imposed by loop closures can be simply modeled by Lagrange multipliers. Along these two tracks, Li et al. [19] developed a dynamic model for the optimal design of a 2-DOF planar parallel robot by maximizing the mean value of the first order natural frequency subject to the prescribed force/motion transmissibility. Piras et al. [20] formulated the dynamic model of a 3-RRR planar parallel mechanism by which the variation of lower natural frequencies with the system configurations were discussed. Zhou et al. [21] studied the vibration problem of a 3-RPS parallel manipulator by treating the joints as virtual springs. Fattah and Angeles [22] investigated the influence of link compliances on the positioning accuracy of a 3-DOF parallel robot, and Wang and Mills [23] discussed the control issue of a 3-PRR flexible planar parallel mechanism. In order to improve the computational efficiency without losing too much accuracy, substructure synthesis or component mode synthesis (CMS) method [24] would be a good choice in dealing with dynamic modeling of mechanical systems that contain movable components (links) having complex geometry. The merit of this method lies in that the number of degrees of freedom (DOF) of a component substructure (link) precisely modeled by FE can significantly be reduced by modal reduction technique while keeping the essential static and dynamic behaviors of the system almost unchanged. The CMS method has been employed, through case-by-case studies, to deal with dynamic modeling of four-bar linkages [25] as well as simple serial or parallel kinematic chains [26]. For example, by taking a two-link serial kinematic chain as an example, Liew et al. [25] proposed a mixed-interface substructure synthesis method for dynamic analysis of multibody systems. In comparison with the results obtained by FEA method, they showed that the computational time can be significantly saved subject to a given accuracy. More recently, Law and Altintas [27] studied dynamic modeling problem