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Research paper

Kinematics and dynamics of a 4-PRUR Schönflies parallel manipulator by means of screw theory and the principle of virtual work

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ABSTRACT

In this work the kinematics and dynamics of a 4-PRUR parallel manipulator with actuated prismatic joints performing the 3T1R motion are investigated by means of the theory of screws and the principle of virtual work. The loss rotations of the moving platform are easily explained by determining and elucidating the effect of two independent constraint couples. Meanwhile, the input-output equations of velocity and acceleration of the robot are obtained in compact form by a systematic application of the properties of exclusive reciprocal screws. To this aim, in order to generate full rank Jacobian matrices, four pseudo revolute joints *connecting* the limbs to the fixed platform are added to the robot. The dynamic analysis of the robot is reported through the formulation of the generalized forces affecting the motion of any body of the parallel manipulator based on a harmonious combination of the theory of screws and the principle of virtual work. A case study is included with the purpose to exemplify the viability of the method.

1. Introduction

Three independent translations and one rotation about one axis of fixed orientation are the characteristics of the so-named Schönflies motions [1]. These movements are typically produced by the so-called SCARA robot, term coined by Makino and Furuya [2], and over the last years have been successfully implemented in parallel manipulators [3] outstripping the high speed of the SCARA-open-type architecture, exhibiting superior rigidity and accuracy, confirming the brilliant idea introduced almost three decades ago of using parallelograms in robot manipulators [4]. After the formidable contribution of Pierrot and colleagues [3], the proposal of novel 3T1R parallel manipulators with better characteristics with the purpose of reaching the robotics market is a formidable task. In that concern is laudable the effort devoted to introduce different topologies able to perform Schönflies motions [5–9].

The kinematics and dynamics of 3T1R parallel manipulators have been extensively investigated, e.g., optimal workspace and dexterity [10], singularity analysis by using Grassmann geometry [11], forward displacement analysis by means of dual quaternion algebra [12], optimal design based on dynamic performance indices endowed with angular constraints amongst limbs [13], redundantly actuated robots [14,15], elastodynamic optimization [16], rotational high-speed pick-and-place manipulation robots [17] or with reduced number of legs [18,19], reconfigurable parallel manipulators [20], type synthesis of

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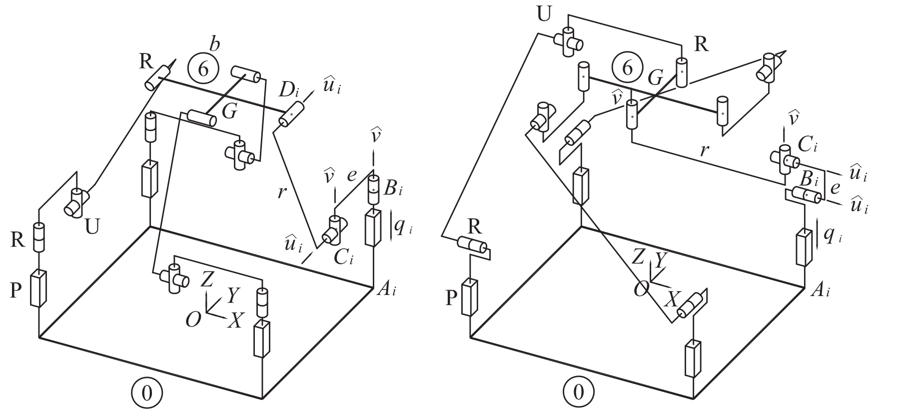

Fig. 1. Two Schönflies parallel manipulators: 4-PRURh left mechanism, 4-PRURv right mechanism.

fully-decoupled 3T1R parallel manipulators by resorting to the theory of screws [8], workspace and singularity analysis of an asymmetrical Schönflies parallel manipulator based on its inverse kinematics [21], stiffness analysis of 4-PRUR parallel manipulators based on screw theory and strain energy [22] and so on. However, among Schönflies parallel manipulators there is one that particularly out stands for its complicated forward kinematics due to the necessity to calculate intermediate variables: the 3T1R parallel manipulator assembled with PRUR-type limbs.

In this work the kinematics and dynamics of a 4-PRUR parallel manipulator equipped with linear actuators and upper revolute joints with axes normal to the plane of the moving platform performing the Schönflies motion, a robot introduced by Kong and Gosselin [23], are investigated by means of the theory of screws and the principle of virtual work, as far as the authors are aware an unpublished subject. This robot is very close to the Isoglide4-T3R1 parallel manipulator proposed by Gogu [24], Rizk et al. [25] and Rat et al. [26], who demonstrated experimentally and theoretically that this kind of robots are more sensitive to leg orientation than to actuator orientation. The rest of the contribution is organized as follows. Once the topology of the robot is outlined in “Description of the robot” section, the kinematic analysis of the robot is approached in Section 3. In that concern, the theory of screws becomes an essential mathematical resource in approaching essential topics in the contribution, e.g., the mobility and loss rotations of the moving platform of the Schönflies parallel manipulator are explained elucidating the effect of two independent constraint couples in “Mobility analysis” subsection. Afterward, the input-output equation of velocity of the robot is obtained in compact form by a systematic application of the properties of exclusive reciprocal screws in “Velocity analysis” subsection. With the purpose to generate full rank Jacobian matrices for the legs of the parallel manipulator, four pseudo revolute joints *connecting* the limbs to the fixed platform are added to the robot. Furthermore, following the trend of the velocity analysis, the section “Kinematics” finishes with the inclusion of the acceleration analysis of the robot, still in our days a controversial issue in the kinematician community when the theory of screws is employed. For the sake of completeness, the kinematic constraint equations associated to the forward-inverse displacement analysis of the robot are easily formulated upon the coordinates of two points embedded in the moving platform in “Displacement analysis” subsection. It is worth to note that this strategy allows to find all the algebraic solutions associated to the solution of the forward displacement analysis, a challenging task of the robot at hand due to the necessity to compute intermediate variables. Finally, the dynamic analysis of the parallel manipulator is achieved by formulating the generalized forces of the parallel manipulator based on a harmonious combination of the theory of screws and the principle of virtual work in “Dynamics” section. Numerical examples are included in order to show the versatility and viability of the method.

2. Description of the robot

Fig. 1 shows two Schönflies parallel manipulators with PRUR-type limbs where the common direction of the axes of the upper revolute joints, horizontal or vertical, makes the main difference between both topologies.

The forward displacement analysis of the robots of Fig. 1 was investigated by Varshovi-Jaghargh et al. [27] based on Study’s parameters applied in seven-dimensional kinematic space. Furthermore, the instantaneous kinematics of the 4-PRURh mechanism was approached by means of screw theory by Gallardo-Alvarado and Garcia-Murillo [28]. To the best knowledge of the authors the instantaneous kinematics and dynamics of the 4-PRURv robot have not been approached by means of the theory of screws and the principle of virtual work in previous works. Thus, hereafter the contribution is devoted to investigate the kinematics and dynamics of the 4-PRURv Schönflies parallel manipulator.

The Schönflies parallel manipulator chosen for the analysis, see Fig. 2, consists of a square moving platform of side b, labeled 6, connected to a square fixed platform of side a, labeled 0, by means of four identical limbs of the type prismatic, revolute, universal and revolute, starting from the fixed platform and ending in the moving platform where the prismatic joints are actuated. In order to explain the geometry of the robot let O_XYZ be a reference frame with associated unit