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Research paper

Extending the capabilities of robotic manipulators using trajectory optimization

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ABSTRACT

The payload capacity of robotic manipulators is often considered to be the same throughout their workspace. However, the actual capacity largely depends on posture, velocity, acceleration and actuator limits. This work studies a method to increase the payload capacity of manipulators through trajectory optimization. This optimization is performed on a task basis and therefore, the load-carrying capacity varies from one task to another. Although the studied method is general and is not limited to specific robot architectures, an analysis of the method is conducted based on its application to a planar RR serial manipulator in a vertical plane. This manipulator is the most appropriate as a simple test case because most manipulators are built in such a way that most of the vertical motion of the manipulator is done by two parallel revolute joints: planar RR mechanism. Simulation and experimental results show that the payload capacity can be greatly increased compared to nominal values. It is also shown that, although the trajectories produced are not time optimal, the method is much more versatile and much simpler to implement than some other optimal control methods. The accompanying video provides a summary of the method and the results.

1. Introduction

The payload capacity of robotic manipulators is often considered to be constant throughout their workspace. One of two assumptions is often made; either that there is no significant change in payload capacity from one configuration to another; or that the payload capacity in the worst case is sufficient to accomplish a given task. For some manipulators, such as industrial robots with high reduction ratios in all joints, these are reasonable assumptions in practice. However, such assumptions tend to lead to very large and massive robots with relatively low payload capacities. In these manipulators, a large portion of the effort supplied by their actuators is used simply to support the large weight of subsequent actuators.

Much like a person cannot hold the same weight with a fully extended arm as closer to his or her body, a robot's actual payload capacity is configuration dependent. Moreover, the true payload capacity, especially for manipulators with low gear ratios, can be greatly influenced by the entire dynamics including velocity and acceleration.

In applications such as human-robot cooperation, humanoid robotics, and other applications, low reduction ratios may provide many advantages. For example, some applications require a high degree of backdrivability or high speed. Such applications are not well suited to joints with large gear ratios. The drawback of this is often a reduction in the joints' force/torque capabilities which limit the payload capacity. Another type of application that could benefit from the methods

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studied in this work are applications where the tasks required are dynamic and are subject to frequent changes. In such cases, an easy to implement method that finds a trajectory that respects many non-linear constraints would be invaluable.

Thus, the goal of this paper is to explore the gains in payload capacity that can be achieved through motion optimization. Two objectives are targeted specifically: the methods must be easily and quickly implementable, and the methods must extend the capabilities of the manipulator in question by finding smooth trajectories that are experimentally feasible.

The motion optimization problem can be divided into two categories: geometric path following and trajectory planning with no prescribed path [1]. Geometric path following has been studied by many researchers with a calculus of variations approach. The calculus of variations applied to the motion of robotic manipulators is optimal control and often makes use of *Pontryagin's maximum principle* [2]. Solutions to the optimal control problem, especially the time-optimal control problem, for the motion of robotic manipulators following a prescribed path have been developed mostly in the mid 1980s [3–5]. An excellent review and explanation of such methods is presented in [6].

These methods are able to take into account the coupled and nonlinear equations of motion to generate a truly optimal path following trajectory. However, it was shown that at each point in a time-optimal trajectory, at least one of the control inputs, torque or force, is at a limit [7]. This results in trajectories that are difficult to follow in practice because such trajectories switch from maximum acceleration to maximum deceleration instantaneously (sometimes at multiple points in a single trajectory). This can cause heavy wear in a manipulator's joints.

In order to alleviate some of the practical problems of the aforementioned optimal control methods, many researchers have approached the geometric path following problem from other angles. Notably, by ensuring that the trajectory planner produces smooth trajectories by using parametric curves [8–10]. These methods are necessarily sub-optimal but produce smooth trajectories.

Geometric path following can include dynamic constraints and can result in great manipulator performances but the problem of finding a geometric path is very difficult. For certain tasks such as arc welding, and machining, a geometric path is naturally imposed by the task. However, in the case where increasing the payload capacity is desired, it is conceivable, even probable, that there are many paths that would be impossible to follow even with the best optimization methods. It is therefore clear that in order to succeed in significantly increasing the payload capacity of a manipulator using it's dynamic capabilities, one must be able to modify both its path and its trajectory which is the second category of motion optimization problems.

As with geometric path following, many researchers have approached the problem from an optimal control perspective by invoking *Pontryagin's maximum principle*. A few hybrid methods were developed [11,12] that rely on solving the geometric path following optimal control problem. These methods start by defining a geometric path using a parametric curve, then finding the optimal control law for following that path. The methods then change the path and iterate until a suitable path and trajectory are found. These methods produce optimal trajectories but not for optimal paths and produce the same type of high jerk trajectories as the path following optimal control methods.

Another approach is to apply Pontryagin's maximum principle directly, not on a specific path. Methods of this kind have been applied to many types of nonlinear control systems, such as the control of fighter jets [13]. In fact, any system of time varying nonlinear differential equations such as the Van der Pol equations can be studied in this way [13]. Extending these methods to systems of nonlinear differential equations with multiple inputs is not simple but can be done [14].

It can be shown through Pontryagin's maximum principle that solutions to the time optimal control problem of systems such as robotic manipulators are of type bang-bang [15]. Indeed, many researchers have developed methods that either force or converge towards bang-singular-bang trajectories for robotic manipulators [16–19]. Again, problems with the premature wear of robotic joints can occur due to the bang-bang nature of these solutions. Some have tried to smooth the trajectories by imposing constraints on jerk or the derivative of the control inputs [20].

The fact that time optimal trajectories are of type bang-singular-bang is a *necessary* condition but not a *sufficient* one, *i.e.*, not all bang-bang trajectories are optimal. As such, finding an optimal bang-bang trajectory is not trivial and requires, in the case of most robotic manipulators, computing the numerical integration of the equations of motion several times which is computationally very intensive.

Grid based combinatorial optimization methods have also been applied to the motion optimization problem [17,21,22], but these methods have been determined to be either prohibitively computationally intensive or to produce non-optimal trajectories.

One interesting approach is to guarantee smoothness by generating inherently smooth parametric joint trajectories. The kinematic and dynamic constraints that ensure the feasibility of a resulting trajectory can be imposed in the optimization process. Many methods have been developed that consider only kinematic constraints such as maximum velocity, acceleration and even jerk [23,24]. For many industrial manipulators, it can be reasonably assumed that the maximum acceleration that each actuator can produce, either changes little as a function of the manipulator's configuration, or that the least maximum acceleration is sufficient for the given task. When trying to increase the payload capacity of a manipulator, these assumptions on the kinematic constraints can be quite restrictive.

Due to the coupled and nonlinear nature of the equations of motion of most robotic manipulators, imposing effort (force or torque) constraints on the generation of trajectories is more difficult than imposing kinematic joint constraints. A gradient based method for optimizing the motion of robotic manipulators was developed in [25], where cubic B-splines were used to define the trajectory of each joint. Dynamic constraints are then imposed on a finite number of points along this trajectory. This methodology is very similar to the one used in this work. Others have extended this method by considering so-called