Mechanism and Machine Theory 121 (2018) 598–612

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Synthesis method for the design of variable stiffness components using prestressed singular elastic systems

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ARTICLE INFO

Article history:
Received 8 April 2017
Revised 25 October 2017
Accepted 17 November 2017
Available online 1 December 2017

Keywords:
Variable stiffness
Antagonistic stiffness
Prestressed elastic systems
Synthesis method

ABSTRACT

The design of variable stiffness components is of interest in several applicative contexts. One way to provide large stiffness variation is to use antagonistic stiffness in prestressed elastic systems composed of linear springs. Interestingly, such systems can be designed so that the stiffness in specific directions is only controlled by the prestress within the springs. In the absence of an adequate synthesis method, their exploitation for this purpose however relies today solely on the designer ability to find an arrangement of the springs that meet his requirements in terms of antagonistic stiffness variation. In this paper, a method is introduced for the synthesis of variable stiffness components using prestressed elastic systems. This method takes into account the antagonistic stiffness coming from the prestress and thus provides an efficient way to meet user-defined requirements. Several synthesis problems for usual variable stiffness components are assessed. It shows the effectiveness of the method to provide new arrangements suitable for implementations. The ability of the method to identify alternate arrangements for a given problem is also shown to ease the design, notably by introducing an exploration strategy using a predictor-corrector method.

1. Introduction

The design of variable stiffness components is of importance in several contexts, ranging from semi-active vibration reduction [1] to compliant robotics, for which several design strategies of variable stiffness actuators are based on such components [2].

In absence of external load, the configuration of such component is in equilibrium and stable. An external load applied on the component will create a small displacement, and its stiffness is defined as the ratio of the external load to the infinitesimal displacement it creates [2]. A variable stiffness component is then defined as a mechanical system whose stiffness can be modulated in pre-defined directions while keeping unmodified its unloaded equilibrium configuration, reached in the absence of external load acting on the system, as considered in [1]. In the following, we consider only infinitesimal

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https://doi.org/10.1016/j.mechmachtheory.2017.11.013

Nomenclature

$\wedge$	cross product operator
a	level of prestress
c	small scalar
m	number of independent inextensional mechanisms
n	number of springs
q	number of disturbance modes
s	number of independent states of self-stress
r	rank of the elastic stiffness matrix
$\mathcal{B}$	base of the elastic system
$\mathcal{M}$	m-dimensional manifold of the inextensional mechanisms
$\mathcal{M}_i$	i-th independent unitary inextensional mechanism
$\mathcal{N}$	nullspace of the non-linear system Jacobian matrix
$\mathcal{P}$	platform of the elastic system
$\mathcal{R}$	non-linear system
$\delta\mathcal{R}$	residual of the non-linear system
$\mathcal{S}$	s-dimensional manifold of the prestress sets
$\mathbf{p}$	parameter vector
$\delta\mathbf{p}$	disturbance of the parameter vector
$\mathbf{r}_i$	spring anchoring distance vector
$\mathbf{s}_i$	spring axis unit vector
$\boldsymbol{\tau}$	springs tension vector
$\mathbf{S}_i$	spring screw
τ_i	spring tension
l_i, l_{i0}	spring length, free-length
k_i	spring axial stiffness
$\mathbf{J}$	Jacobian matrix of the non-linear system
$\mathbf{K}_e$	elastic stiffness matrix (6×6)
$\mathbf{K}_a$	antagonistic stiffness matrix (6×6)
$\boldsymbol{\Omega}$	springs axial stiffness matrix $(n \times n)$
$\mathbf{N}_{\mathcal{S}}$	states of self-stress matrix $(n \times s)$
$\mathbf{N}_i$	i-th disturbance mode
$\mathbf{S}$	unitary screws matrix $(6 \times n)$

displacement under external load, so that the behavior of the system can be linearized about this configuration to describe its behavior by a stiffness matrix.

There are several design strategies of variable stiffness components. One of them is the use of prestress elastic systems which has been reported to be particularly relevant to design variable stiffness component in this context [3]. Generally speaking, elastic systems are constituted by a set of springs that relates a rigid platform to a rigid base [4] (Fig. 1). Their lightweight nature and their proximity with bioinspired mechanisms [5,6] make them of interest. The stiffness of the coupling between the base and the plateform partly depends on the *antagonistic stiffness* coming from the level of internal forces within the springs, namely the *prestress*. The use of the prestress modulation to generate variable stiffness can be very effective to obtain large stiffness variations without modifying the equilibrium configuration [1].

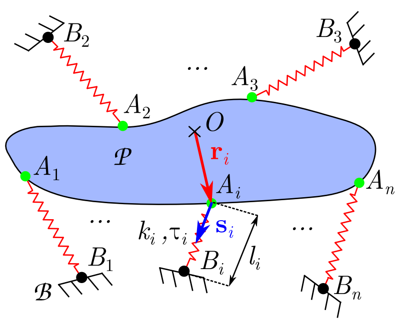

Fig. 1. General structure of elastic system. The platform $\mathcal{P}$ is connected to the base $\mathcal{B}$ by means of n springs.