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A three-step methodology for dimensional tolerance synthesis of parallel manipulators

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ABSTRACT

Computing the maximal pose error given an upper bound on model parameters uncertainties, called perturbations in this paper, is challenging for parallel robots, mainly because the direct kinematic problem has several solutions, which become unstable in the vicinity of parallel singularities. In this paper, a local uniqueness hypothesis that allows safely computing pose error upper bounds using nonlinear optimization is proposed. This hypothesis, together with a corresponding maximal allowed perturbation domain and a certified crude pose error upper bound valid over the complete workspace, will be proved numerically using a parametric version of Kantorovich theorem and certified nonlinear global optimization. Then, approximate linearizations are used in order to determine approximated tolerances reaching a prescribed maximal pose error over a given workspace. Those tolerances are finally verified using optimal pose error upper bounds, which are computed using global optimization techniques. Two illustrative examples are studied in order to highlight the contributions of the paper.

1. Introduction

For two decades, parallel manipulators have attracted the attention of more and more researchers who consider them as valuable alternative design for robotic mechanisms. Parallel Kinematics Machines (PKM) offer essential advantages over their serial counterparts such as lower moving masses, higher stiffness and payload-to-weight ratio, higher natural frequencies and better dynamic performance.

However, PKM are not necessarily more accurate than their serial counterparts. Indeed, even if the dimensional variations can be compensated with PKM, they can also be amplified contrary to with their serial counterparts. Wang et al. [1] studied the effect of manufacturing tolerances on the accuracy of a Stewart platform. Kim et al. [2] used a forward error bound analysis to find the error bound of the end-effector of a Stewart platform when the error bounds of the joints are given, and an inverse error bound analysis to determine those of the joints for the given error bound of the end-effector. Kim and Tsai [3] studied the effect of misalignment of linear actuators of a three Degree-of-Freedom (DOF) translational parallel manipulator on the motion of its moving platform. Han et al. [4] used a kinematic sensitivity analysis method to explain the gross motions of a 3-UPU parallel

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mechanism, and showed that it is highly sensitive to certain minute clearances. Fan et al. [5] analyzed the sensitivity of the 3-PRS parallel kinematic spindle platform of a serial-parallel machine tool. Verner et al. [6] presented a new method for optimal calibration of PKM based on the exploitation of the least error sensitive regions in their workspace and geometric parameters space. As a matter of fact, they used a Monte Carlo simulation to determine and map the sensitivities to geometric parameters. Moreover, Caro et al. [7] developed a tolerance synthesis method for mechanisms based on a robust design approach. Ryu et al. derived a volumetric error model and a total error transformation matrix from a differential inverse kinematic equation, which includes all kinematic error sources [8]. Liu et al. reported an approach of geometric error modeling for lower mobility manipulators by explicitly separating the compensatable and uncompensatable error sources affecting the pose accuracy [9]. Briot and Bonev proposed a simple method based on a detailed error analysis of 3-DOF planar parallel robots that brings valuable understanding of the problem of error amplification [10]. Rolland used algebraic tools in order to compute an upper bound of the moving platform pose error for a Gough-Stewart platform while considering geometric and numerical errors [11–13]. Merlet and Daney used a first order approximation of the pose error to assess the pose error in a workspace and perform a dimensional synthesis of the manipulator [14]. Patel and Ehmann analyzed the volumetric error of a Gough-Stewart platform too [15].

During the early design process of engineering systems, the analysis of the performance sensitivity to uncertainties is an important task. High sensitivity to parameters that are inherently noisy can lead to poor, or unexpected performance. For that reason, it is important to analyze the sensitivity of their performance to variations in their geometric parameters and to determine the optimal dimensional tolerances.

To this end, some indices such as the dexterity and the manipulability have been used to evaluate the sensitivity of robots performance to variations in their actuated joints [16–18]. However, they are not suitable for the evaluation of this sensitivity to other types of uncertainty such as variations in geometric parameters. Two indices were proposed in [19] to evaluate the sensitivity of the end-effector pose (position + orientation) of the Orthoglide 3-axis, a 3-DOF translational PKM, to variations in its design parameters. In the same vein, four 3-RPR planar parallel manipulators (PPMs) were compared in [20] based on the sensitivity of their performance to variations in their geometric parameters. In [21], an interval linearization method is used for the sensitivity analysis of some parallel manipulators. However, the foregoing research works do not deal with the tolerance synthesis of parallel manipulators, which is a critical issue.

In the present paper, we overcome two lacks in the literature: First, a fully rigorous methodology is proposed to compute a certified upper bound for the pose error due to bounded uncertainties in the model parameters of a PKM throughout its workspace. Second, a methodology is proposed for the tolerance synthesis of PKM, aiming at synthesizing the largest tolerances while keeping the pose error of the moving-platform below a given limit. The proposed tolerance synthesis method is composed of three steps:

- Step 1 A rigorous parametric pose error upper bound $\epsilon(\mathbf{p})$ is computed, which depends on the value of the perturbation $\mathbf{p}$, together with a perturbation domain $\mathcal{P}$ where this upper bound is valid. Both are computed using Kantorovich theorem, where Kantorovich constants are evaluated over the full manipulator workspace using certified nonlinear global optimization.
- Step 2 Since the previous upper bound is pessimistic, its usage for tolerance synthesis may lead to some over design (i.e., too small tolerances are designed leading to a better accuracy than the required one). Therefore, a non rigorous linearization of the maximum pose error in the workspace is proposed and used for synthesizing approximate tolerances.
- Step 3 A rigorous sharp pose error upper bound is finally computed for the tolerance synthesized at Step 2 using certified nonlinear global optimization. The cruder upper bound computed at Step 1 is necessary to make this problem provably consistent.

Step 2 is actually optional: An accurate linear approximation can be obtained by building a linear model using the sharp error upper bound computed at Step 3 for different tolerances. However, the problem to be solved at Step 3 is more difficult than ones to be solved at Step 2, therefore starting with the linear approximation provided by Step 2 can turn out to be more efficient.

The paper is organized as follows. Step 3 motivates the necessity of Step 1, and is therefore first detailed in Section 3. A uniqueness hypothesis is introduced in order to compute a certified sharp pose error upper bound over a given workspace by solving a nonlinear optimization problem. Step 1 is the central contribution of the paper, and is addressed in Section 4: A parametric version of Kantorovich theorem is proposed, which provides both a maximal perturbation domain for which this uniqueness hypothesis holds, and a crude certified pose error upper bound valid inside this perturbation domain as well as in the manipulator workspace. Step 2 is finally developed in Section 5: An approximate linearization of the maximal pose error of the moving-platform in the workspace is proposed, which allows performing some approximate tolerance synthesis. These approximate tolerances can finally be corrected using the results obtained in Section 3. Two illustrative examples are given in Section 6 in order to highlight the potential and limits of the approach. The two examples deal with the tolerance synthesis of a R$\underline{\text{P}}$R$\underline{\text{P}}$R parallel manipulator and a 3–R$\underline{\text{P}}$R parallel manipulator with a fixed orientation of this moving-platform, respectively.