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Forced vibrations of a marine propulsion shafting with geometrical nonlinearity (primary and internal resonances)

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ABSTRACT

In this paper, the longitudinal primary resonance of a marine propulsion shafting is investigated with special consideration to the case with an internal resonance (the first longitudinal natural frequency is approximately equal to the sum of the first transverse forward and backward frequencies). Coupled longitudinal-transverse dynamic equations of a marine propulsion shafting are established by the Ritz method and the Lagrange equation. Then these equations are solved by the method of multiple scales. The steady-state response and the stability are analyzed. Research shows that the first transverse forward and backward modes could be excited if the longitudinal excitation load is larger than a critical load. There is saturation phenomenon in the longitudinal motion and the extra energy is transferred to the transverse mode. The energy distribution ratio between the forward and backward modes is inversely proportional to their frequency ratio. At last, the effects of damping ratio and frequency detuning parameters on the critical load are discussed. Results of perturbation method are validated by numerical simulations.

1. Introduction

Propulsion shafting is an important unit in the marine components. Dynamic analyses of propulsion shafting are essential for marine design engineers. The propulsion shaft system could be considered to be a typical rotor-bearing system. Early, dynamic studies on rotating shafts focused on linear vibrations, including the prediction of the natural frequencies, calculation of the unbalanced response and so on [1–3]. For some structures, a linear dynamic analysis model may be sufficient when the vibration response is not large. However, the linear theory is not valid any more when the amplitude of oscillation or the divergence is sufficiently large, and one has to resort to nonlinear models. Most often the type of nonlinearity is geometric coming from the nonlinear relation between strains and displacements. For propulsion shafts, the propeller (the mass may be up to a few tens of tons) is very large and has inescapable eccentricity due to machining accuracy or wear and tear. Therefore, it suffers from a huge unbalanced excitation when it rotates. At the same time, it also suffers from a very large fluid pulse pressure in the longitudinal direction. Hence, the vibration amplitudes are very large and the nonlinear coupling between transverse deflection and longitudinal deflection is easy. Therefore, it's meaningful to study the nonlinear dynamic response. The recent literature related to nonlinear vibration of beams or shafts is quite large. These studies can be categorized mainly into three general classes in terms of models being considered [4].

In the first class, only transverse motion of rotor is considered and longitudinal displacement is neglected. For example, Rizwan [5] developed a mathematical model incorporating higher order deformations in bending and analyzed nonlinear dynamics of rotors. Ishida and coworkers in several papers considered nonlinear vibrations of the Jeffcott rotor which the nonlinear

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characteristics were approximated by the second and the third power terms of coordinates. They studied nonlinear forced oscillations of a rotating shaft with quadratic nonlinearity in the restoring force [6]. Furthermore, they investigated the entrainment phenomena at the critical speeds of 1/2 order subharmonic oscillations of forward and backward whirling modes [7]. They also studied nonstationary vibrations [8] and internal resonances [9,10] of a nonlinear rotating shaft system which the forward natural frequency and the backward natural frequency satisfied the internal resonance relation of $1{:}-1$.

In the second class, both longitudinal and transverse displacements are considered, and coupled partial differential equations of motion are obtained. Then it is assumed that the longitudinal displacement is a function of the transverse displacement by neglecting the longitudinal inertia or making extensional or in-extensional assumption [11]. To reduce the degree of freedom, then the final coupled partial differential equations were transformed into the form of integro-partial differential equations by substituting this function relation into the transverse equations. Most of the studies regarding this topic can be grouped into the second class. For example, Dwivedy [12–16] examined nonlinear dynamics of a slender beam carrying a lumped mass. These analyses included the principal parametric resonance, combination resonance and internal resonance. Nayfeh [17] investigated nonlinear normal modes of a fixed-fixed buckled beam consideration of the three-to-one and one-to-one internal resonances. Emam analyzed nonlinear responses of buckled beams to 1:1 and 3:1 internal resonances [18] and subharmonic resonances [19]. Barari [20] studied nonlinear vibrations of Euler-Bernoulli beams subjected to the axial loads by variational iteration and parametrized perturbation methods. The foregoing analyses relate to the plane beam structures. For the rotating shafts, there are also some studies. Ishida discussed forced oscillations of a vertical continuous rotor with geometric nonlinearity due to the extension of the rotor center line [21]. Khadem and coworkers made outstanding achievements about the nonlinear rotating shafts. They discussed primary and parametric resonances of asymmetrical rotating shafts with stretching nonlinearity [22] and primary resonances of a nonlinear in-extensional rotating shaft [23]. They also investigated two-mode combination resonances of a simply supported rotating shaft by the method of harmonic balance [24]. Hosseini and coworkers also did lots of work in this field. They analyzed free vibrations of a rotating shaft with nonlinearities in curvature and inertia [25] and with stretching nonlinearity [26]. They investigated primary resonances of a rotating shaft with stretching nonlinearity [27]. Furthermore, they studied the dynamic stability and bifurcations of a nonlinear in-extensional rotating shaft with internal damping [28]. In addition, Shaw [29] studied dynamic responses of an unbalanced rotating shaft with internal damping using the center manifold approach. Luczko [30] investigated dynamic responses of a rotating shaft with internal resonances and self-excited vibrations. Nagasaka [31] discussed forced oscillations in the vicinities of both the major and the secondary critical speeds of a continuous asymmetrical rotor with geometric nonlinearity.

In the third class, both longitudinal and transverse displacements are considered and the corresponding equations of motion are obtained. In this class the two coupled partial-differential equations are solved together with no addition assumptions. For example, Han [32,33] investigated free vibrations and forced responses of a compliant tower with consideration of the coupled transverse and axial motion by the finite difference approach, considering the longitudinal inertia. Ghayesh [4] studied coupled longitudinal-transverse dynamics of an axially moving beam with and without a three-to-one internal resonance between the first two transverse modes. He investigated nonlinear vibrations and stability with an intermediate spring support of an axially moving beam [34] and Timoshenko beam [35]. He analyzed nonlinear dynamics of an axially accelerating beam [36]. He also analyzed in-plane and out-of-plane motion characteristics of microbeams with one-to-one internal resonance between the in-plane and out-of-plane transverse modes [37]. These analyses both relate to the beams and the literature on the nonlinear vibration of rotating shafts categorized in the third class is very few.

To sum up, some studies of nonlinear beam or rotating shaft vibrations are based on the in-extensional assumption. It may be reasonable for no external loads acting along longitudinal direction. And some studies are based on the simplification that the longitudinal inertia is small and it follows that $u = o(w^2)$. It may be reasonable for the slender simple support rotors (beams) without disks because the first longitudinal frequency is much large than the first lateral frequency [38]. For propulsion shafting, the longitudinal inertia is not negligible due to the existence of the propeller mass, and the first frequency can be a stretching mode. Hence, the longitudinal inertia and the restoring force may be the same order and it follows that $u = o(w)$. Therefore, in this paper, the longitudinal primary resonance of a marine propulsion shafting is investigated with special consideration to the internal resonance between the longitudinal and transverse directions for the first time. For some shafts, the first longitudinal natural frequency may be equal to the sum of the first transverse forward and backward frequencies ($\omega_u \approx \omega_f + \omega_b$) and hence, the longitudinal and transverse modes may interact with each other due to internal resonances. A coupled longitudinal-transverse dynamic model of the marine propulsion shafting is established due to stretching nonlinearity. The shaft is assumed to be supported by some linear-elastic springs and with a lumped mass at one end. Rotary inertia and gyroscopic effect are included, but shear deformation is neglected. Coupled longitudinal-transverse dynamic equations are derived by the Ritz method and the Lagrange equation. Then these equations with gyroscopic terms are solved by the method of multiple scales. The steady-state response and the stability are analyzed. The effects of damping ratio and frequency detuning parameters on internal resonances are discussed. Results of perturbation method are validated with numerical simulations.

2. Mathematical model

The main structures for most of the marine propulsion shafting are similar. A schematic diagram for a typical marine propeller shafting is illustrated in Fig. 1. It consists of a shaft, propeller, back stern bearing, front stern bearing, middle bearing and thrust bearing.

Because the shaft links with a motor by a flexible coupling with small stiffness, the shaft and the motor could be separated. The shaft is assumed to be a uniform cross-section beam; the propeller is considered to be a lumped mass; and the bearings are considered to be linear springs. On the basis of these assumptions, the propulsion shafting is simplified as shown in Fig. 2.