Mechanism and Machine Theory 105 (2016) 337–351

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Synthesis of eight-bar linkages by constraining a 6R loop

K.H. Sonawale*, J.M. McCarthy

Robotics and Automation Laboratory, Mechanical and Aerospace Engineering, University of California, Irvine, CA 92697, USA

ARTICLE INFO

Article history:
Received 10 April 2015
Received in revised form 29 June 2016
Accepted 2 July 2016
Available online xxxx

Keywords:
Kinematic synthesis
Eight-bar linkage
Linkage design

ABSTRACT

This paper presents a design system for planar eight-bar linkages that begins with a user specified 6R planar loop and five required configurations, and computes two RR constraints that yield an eight-bar linkage. There are 32 ways that these constraints can be added to the 6R loop to yield as many as 340 different linkages, which include eight of the 16 eight-bar linkage topologies. An analysis routine based on the Dixon determinant is used to verify the performance of each design candidate. Random variation of task configurations within user specified tolerance zones is used to increase the number of candidate designs. The result is an effective system for the design of eight-bar linkages, which is demonstrated by designing linkages that guide movement though a symmetric and offset set of parallel task positions along a straight line.

1. Introduction

This paper presents a design system for eight-bar linkages that adds two RR dyads to a user-specified 6R planar loop shown in Fig. 1. The approach specifies five configurations and then uses five-position synthesis of two RR dyads to constrain the relative movement of links in the 6R loop to obtain an eight-bar linkage.

The design system generates all of the candidate linkages available from the 32 ways the 6R loop can be constrained, and then evaluates their performance to ensure successful designs. The number of design candidates is increased by varying the task requirements within user specified tolerance zones. This system is demonstrated by finding successful eight-bar linkage designs for a set of parallel task positioned on a straight line.

2. Literature review

A design theory for linkages with six, eight and 10 bars was presented by Kempe [1], who provided geometric techniques for the design of 8-bar linkages that trace an exact straight line. Mueller [2] introduced a graphical approach for the synthesis of an eight-bar linkage. Also see the work of Hain [3] and Hamid and Soni [4]. Chen and Angeles [5] developed a method to synthesize an eight-bar linkage obtained by coupling two four-bar linkages that can reach 11 specified task positions.

The use of two RR dyad to transform a 6R loop into an eight-bar linkage was introduced by Soh and McCarthy [6]. Central to this process is the calculation of an RR dyad that connects two relatively moving bodies introduced by Burmester [7], also Sandor et al. [8]. For a discussion of Burmester's work see Koetsier [9].

The designer specifies five task configurations for the 6R loop, and the design system calculates the inverse kinematics of the system to determine the position and orientation every link in the system [10]. The synthesis routine uses a graph theory

* Corresponding author.
E-mail addresses: ksonawal@uci.edu (K. Sonawale), jmmccart@uci.edu (J. McCarthy).

http://dx.doi.org/10.1016/j.mechmachtheory.2016.07.001
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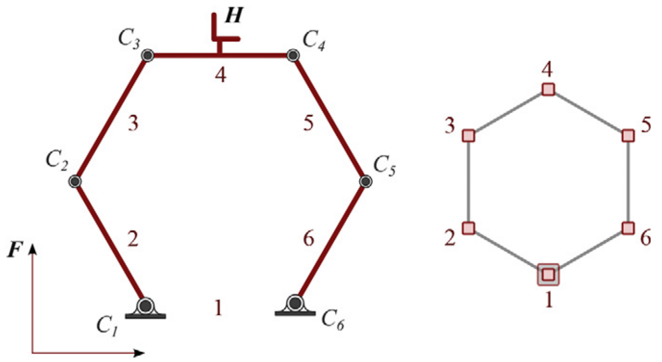

Fig. 1. A 6R loop with hinged joints $C_1, \ldots, C_6$ and links $\{1, 2, \ldots, 6\}$, together with its linkage graph. Notice that the ground is link 1 and the end-effector is link 4.

based procedure to identify the RR dyads that yield eight-bar linkages. This provides an automated way to formulate the design equations for the 32 different cases yielding eight of the sixteen eight-bar linkage topologies [11].

The design system analyzes each candidate design in order to verify performance. This is done using the analysis algorithm developed by Parrish et al. [12], which uses the adjacency matrix of a linkage graph to characterize the design [13].

This paper is an extension of research on eight-bar linkage design system presented in Sonawale and McCarthy [14] and Sonawale [15]. The initial results in this area focused on adding three RR dyads to a 4R serial chain can be found here [16]. Examples show this design system yields a large number of successful designs even for a parallel set of task positions distributed on a straight line.

3. Synthesis of an RR constraint

The usual formulation of Burmester's synthesis equations assumes that the RR constraint connects a moving body M to a fixed body F. This can be generalized for the purposes of this design system, by assuming five positions of the first moving body represented by frames M_j and five positions of the second moving body represented by frames $F_j, j = 1, \ldots 5$, are known. Let the 3×3 homogeneous transformations $[R_j]$ and $[S_j]$ define the position and orientation of M_j and $F_j, j = 1, \ldots 5$, respectively, in the ground frame, given by,

$$[R_j] = \begin{bmatrix} \cos\gamma_j & -\sin\gamma_j & a_j \\ \sin\gamma_j & \cos\gamma_j & b_j \\ 0 & 0 & 1 \end{bmatrix}, \quad [S_j] = \begin{bmatrix} \cos\sigma_j & -\sin\sigma_j & c_j \\ \sin\sigma_j & \cos\sigma_j & d_j \\ 0 & 0 & 1 \end{bmatrix}, \quad j = 1, \ldots, 5. \tag{1}$$

Let $\mathbf{w} = (w_x, w_y, 1)$ be the homogeneous coordinates of point fixed in the frame M and, similarly, let $\mathbf{g} = (g_x, g_y, 1)$ be fixed in F, so

$$\mathbf{W}^j = [R_j]\mathbf{w}, \quad \mathbf{G}^j = [S_j]\mathbf{g}, \quad j = 1, \ldots, 5. \tag{2}$$

The constraint equations for an RR crank that connect the frames M_j and $F_j, j = 1, \ldots 5$, are given by,

$$(\mathbf{W}^j - \mathbf{G}^j)\cdot(\mathbf{W}^j - \mathbf{G}^j) = R^2, \quad j = 1, \ldots, 5, \tag{3}$$

where the dot denotes the usual vector dot product, and R is a constant that defines the length of the RR crank. These five equations can be solved for the coordinates of $\mathbf{w}$ and $\mathbf{g}$ and the length R.

It is convenient to reformat the equations in Eq. (2) so that the coordinates of the RR crank pivots are defined in the ground frame G as $\mathbf{W}^1 = (x, y, 1)$ and $\mathbf{G}^1 = (u, v, 1)$. This is done by introducing the relative transformations,

$$[R_{1j}] = [R_j][R_1]^{-1} \quad [S_{1j}] = [S_j][S_1]^{-1}, \quad j = 1, \ldots, 5, \tag{4}$$

so that

$$\mathbf{W}^j = [R_{1j}]\mathbf{W}^1 \quad \mathbf{G}^j = [S_{1j}]\mathbf{G}^1, \quad j = 1, \ldots, 5. \tag{5}$$