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Short communication

A note on enveloping curves in plane

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ABSTRACT

This paper revisits the classical envelope theory in plane, which deals with two planar curves in a point contact moving in relative roll-slide motion. The well known result about the centers of curvature of the generating curve and the envelope curve behaving as coordinated centers, is shown to be valid even if the instantaneous relative angular velocity is zero. The analytical approach uses the contact kinematics equations and does not require the existence of finitely accessible velocity pole and polodes of the relative motion. An example of equivalent mechanisms demonstrates the extended applicability of the theorem on coordinated centers of enveloping curves.

1. Introduction

Euler-Savary envelope equation [1–3] and the theorem on coordinated centers [2–4], are well known results in geometric kinematics in plane. The envelope equation relates the curvatures of two curves in roll-slide motion and the location of the velocity pole of relative motion, to the relative curvature of the polodes. The theorem on coordinated centers, which is the prime interest of this paper, essentially states that *the center of curvature of instantaneous point-path of the center of curvature of a generating curve coincides with the center of curvature of its envelope*. The proof for this theorem is well reported in literature. In [3], the result is analytically derived by comparing the Euler-Savary envelope equation and the Euler-Savary equation for point-path curvature, which explicitly require reference to finitely located pole and polodes in the plane. Similarly, Dijksman [4] proved the theorem on coordinated centers of enveloping curves through synthetic geometry arguments using the concepts of pole, pole-tangent, pole-velocity, and the Hartmann's line. *All the existing derivations overlook the case when the instantaneous angular velocity of the relative motion is zero*, which means that the polodes for the roll-slide motion of the enveloping curves are not finitely accessible in the plane. Such an instantaneous configuration of the moving plane with respect to the fixed plane is more precisely called T_1*-position of the first-kind* [3,5-7]. At a T_1-position of the first-kind, the instantaneous angular velocity of a moving plane is zero but the translational velocity is non-zero. It is mentioned in [3] that the Euler-Savary equation and the Hartmann's theorem are not recognizable in this position. Since these concepts form the basis of the existing methods, they do not take into consideration the case of T_1-position of the first-kind.

In this paper, we are interested in the case when the relative motion between the two enveloping curves is in the T_1-position of the first-kind. We show that the theorem on coordinated centers is valid even in this position. Our approach uses the contact

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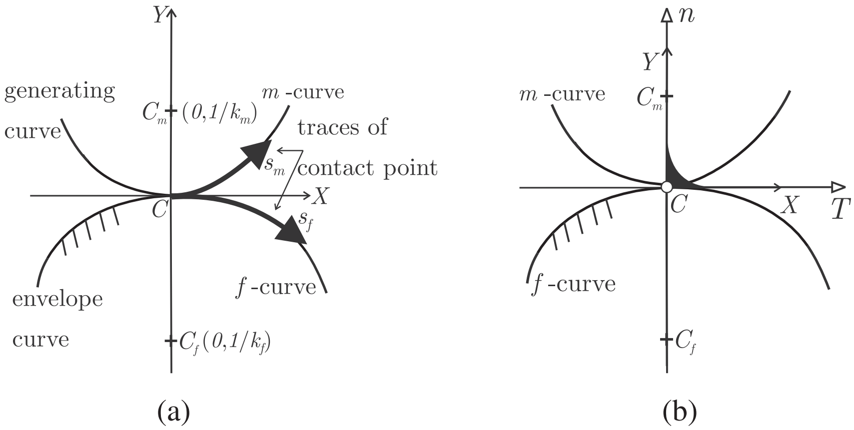

Fig. 1. Geometry of enveloping curves. (a) Generating and envelope curves and trace of contact point along the curves, (b) auxiliary rigid body *Tnp* for kinematic analysis.

kinematics equations of the enveloping curves and does not make any reference to the polodes which makes it valid even when the instantaneous angular velocity is zero. The applicability of the result to an instantaneous translation configuration is used for classical lower pair to higher pair substitute connection in an example of equivalent mechanisms.

2. Contact kinematics and coordinated centers

Contact kinematics deals with the study of relative motion between two bodies in persistent contact over locally smooth geometries on the bodies. Literature seems to consider contact of degenerate local geometry of one of the bodies on the smooth geometry of the other body also as systems of interest. The equations capturing the motion of these systems are called as contact kinematics equations [8]. Studies on contact kinematics of surfaces in point contact and line contact are available in [8,9,10] and [10,11] respectively. The analysis of relative roll-slide motion between two planar curves in a point contact is studied by Cai and Roth [12], and also by Griffis [9] as a special case of 3D surfaces in point contact. The framework presented by Griffis was first applied by Dijksman [4], which uses *an intermediate virtual body* between the contacting bodies. Griffis [9] introduced higher-order reciprocity for two surfaces in point contact where a set of contact kinematics equations are derived for higher-order conjugate motion using *twist* and *twist derivative* in the *dual vectors set up*. In this paper, we use the framework and the contact kinematics equations of Griffis for our analysis.

The basic geometric entities in envelope theory are two curves in a point contact which are in relative roll-slide motion. Without loss of generality, one of them can be assumed to be fixed. The generating curve and its envelope are labeled as the m-curve and f-curve respectively; their point of contact is C (Fig. 1 (a)). We refer to the contact normal line as the n-line (labeled n in Fig. 1 (b)) in this paper for brevity wherein n is assumed to be oriented and its orientation is away from the f-curve, or towards the m-curve. A right-handed coordinate system, XYZ, is attached to the fixed frame with its origin at the point of contact C. The positive Y-axis is chosen along the n-line which in the direction away from the material side of the f-curve at C and the positive X-axis is chosen such that positive Z-axis points out of the plane of the paper. Let the centers of curvature of the f-curve and m-curve be C_f and C_m with coordinates $C_f = [0, 1/k_f]^T$ and $C_m = [0, 1/k_m]^T$ respectively, where k_m and k_f are the signed curvatures of the m-curve and f-curve at C. It may be noted that $(k_m - k_f) > 0$ for all contacting geometries in the chosen convention.

Let $T = \underline{T} + \epsilon\underline{0}$ be the common tangent line at C directed along the instantaneous velocity of the point of the m-curve which coincides with C. Also, $n = \underline{n} + \epsilon\underline{0}$ be the contact normal line (Fig. 1 (b)). Then, the line $p = \underline{p} + \epsilon\underline{0}$ which passes through C is such that Tnp is a *right handed mutually orthogonal triad* of lines. $\underline{T}$, $\underline{n}$, and $\underline{p}$ are unit vectors. A *virtual three-link mechanism* consisting of the f-curve(body-1 in [9]), the m-curve (body-2 in [9]) and Tnp based virtual body (body-3 in [9]) is used to derive the contact kinematics between the two curves. The instantaneous relative velocity between the points of the curves at the point of contact C is directed along $\underline{T}$. Let $\dot{s}_m$ and $\dot{s}_f$ be the speeds with which the point of contact moves along the m-curve and f-curve respectively and v_c be the speed of the point of the m-curve which coincides with C. $S = \underline{S} + \epsilon\underline{S}_o$ and $\dot{S} = \underline{\dot{S}} + \epsilon\underline{\dot{S}}_o$ are the twist and twist derivative dual vectors respectively. The contact kinematics equations given in Equation (10) in [9] are reproduced below for the sake of completeness.

$$0 = \dot{s}_2 t_2 - \dot{s}_1 t_1 + \underline{S} \cdot \underline{T} \tag{1}$$

$$0 = \dot{s}_2 \gamma_2 - \dot{s}_1 \gamma_1 + \underline{S} \cdot \underline{n} \tag{2}$$

$$0 = \dot{s}_2 k_2 - \dot{s}_1 k_1 + \underline{S} \cdot \underline{p} \tag{3}$$