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A linear complementarity formulation for contact problems with regularized friction

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ABSTRACT

A mathematical formulation in terms of a linear complementarity problem with regularized friction is introduced for multibody contact problems. In this approach, contacts are characterized based on kinematic constraints while the friction forces are simultaneously regularized and incorporated into the formulation. The variables of the resulting linear complementarity problem are only the normal forces. The main advantage of this formulation is that the dimension of the resulting linear complementarity problem is significantly less than its counterpart formulations in the literature, and hence, faster simulations can be achieved. The proposed formulation is examined for a set of benchmark examples yielding promising results.

1. Introduction

Contact modelling, simulation, and analysis of mechanical systems need proper treatments due to the non-smooth nature of the problem. Constraint and explicit force representation formulations are two main approaches to deal with the contact problems [11,16,17,28,36]. In constraint based approaches, the contact problem is formulated by imposing the kinematic unilateral constraints between bodies in contact. Such a representation was first formulated in Ref. [30], which was further expanded using methods of modern mathematics such as measure differential equations that may directly lead to effective numerical algorithms for contact problems [29].

For frictionless sustained contacts, a complementarity problem can be formed using the fact that either normal contact velocity or force exists at each time instance. In this case the resulting mathematical model is a linear complementarity problem (LCP) and relatively straightforward to solve, because it can be proven that such an LCP always has a solution [27]. By adding the Coulomb friction, as one of the most common friction models, this phenomenon can be still represented by a complementarity problem but the resulting cone is not a positive orthant and hence, non-linear optimization techniques needs to be considered, such as the Gauss–Seidel or fixed-point iteration methods [5,22,23,31]. Another possibility is to derive a linear complementarity problem based on a polyhedral approximation of the actual friction cone [4,17,43]. In this case, the resulting LCP formulation can be stated at the acceleration [17], velocity [4] or position [43] levels. Acceleration level formulations are not guaranteed to always have a solution. This was one of the motivations to propose an LCP formulation at the velocity-level [4]. The formulation

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arose as a result of applying time-stepping schemes for the system of dynamic equations and constraints, and then forming an LCP. It was shown that with such a technique, the solvability of the resulting LCP can be guaranteed [4]. Although the LCP velocity-level formulation is among the most reliable approaches, the solution set might not always be convex, and hence, some difficulties could still arise [3]. Furthermore, penetration can happen as the unilateral constraints are expressed at the velocity-level and not the position level. As an alternative formulation, the LCP can be expressed at the position-level [43]. Such a formulation can guarantee that no penetration happens as the unilateral constraints are expressed in the position level but the allowable motion of the interacting bodies can be unrealistically restricted [44].

There are other possibilities than the time-stepping schemes in the literature to deal with non-smooth contact dynamics [36]. Event-driven schemes divide the dynamic problem into the smooth and non-smooth events. In such methods, established constraints are identified (event detection), and contact forces are computed in such detected events via solving linear or non-linear complementarity problems [1,37]. There is also a completely different approach to deal with contact dynamics, which is based on the Augmented Lagrangian formulation. In such an approach, the constraints and dynamics formulations are expressed as the so-called *prox* formulation, which could bring computational advantages for solving contact problems [2,36].

Considering the LCP formulations, and regardless of how they are formulated, for the solution a well-known category of approaches is based on the so-called *simplex methods*, which are known as the *direct* or *pivoting methods* as well [14,24,32]. Another group of solvers are based on iterative algorithms. Among those are the projected Gauss–Seidel, non-smooth non-linear conjugate gradient, sequential quadratic programming methods, and Jacobi-based solvers [5,22,23,31].

On the other hand, in the explicit force model approaches, the complementarity conditions disappear by relaxing the normal direction and characterizing the normal force based on an appropriate constitutive relation. A selected friction model is then incorporated into the equations. One of the earliest contact force models is the Hertz model [20]. In this model, the normal force at each contact point is characterized based on a stiffness and power function of the penetration level between the interacting bodies. Different stiffness and power exponent values are suggested for different applications. For instance, there are models for two spheres of isotropic material [18], two cylinders [9], a cylinder and a half space [33], and spur gears [45]. It is shown that the Hertz-based contact models are usually more appropriate for hard materials and low initial impact velocities [18]. For soft materials or high initial velocities strain rate effects need to be considered as well [41]. Therefore, Goldsmith [18] and later on Lankarani and Nikravesh [26] proposed models for the case of permanent penetration in soft material. A main advantage of Hertz model and its extensions is that the material and the geometrical properties are reflected in the model but certain weaknesses exist which stem from the assumption of small contact area, and the absence of energy dissipation consideration in these models [28]. The Kelvin and Voigt approach is one of the first contact models with the consideration of dissipative term [18]. Other contact force models with the dissipation consideration were also proposed in the literature [16,28]. While each of these models could more appropriately capture certain aspects of the contact behaviour, they might introduce numerical complexities and artifacts into the calculations. Although by increasing the stiffness values, the model can more accurately capture the rigidity of the contact surface, stiffer differential equations could be obtained at the end. The stiff differential equations are prone to well-known numerical challenges [42]. For instance, small enough time-steps are usually required to achieve stable results. Also, there are additional parameters in these models that need to be tuned based on the specifications of each problem, which make the methods not very user-friendly.

In this paper, an alternative formulation is proposed, which is based on imposing unilateral constraints for contacts while friction forces are regularized in the formulation. As the Coulomb friction does not have an explicit constitutive representation in the static phase, one approach is to regularize the friction force in this phase. By regularized friction it is meant that the static friction forces are approximated by constitutive relations expressed in terms of the generalized coordinates or velocities. The regularization may be done based on the position related variables (e.g., Bristle [19] and LuGre [12] models) or velocity [15]. Eventually, the friction force depends on either the normal force (in the kinetic phase) or position/velocity of the body and the regularization parameters (in the static phase). Such a friction regularization, with the cost of approximating the friction forces, can make the mathematical formulation of the contact problem at hand numerically simpler. Despite the common approaches that the regularized friction force is usually used together with the compliant force relations in the normal direction [16,28], here the friction regularization is combined with the unilateral constraints in the normal direction. With this representation, the dimension of the resulting complementarity problem is reduced, and hence, the computational cost of the proposed approach is reduced. This could have significant importance, specially for real-time simulation of mechanical systems with large number of contacts. In the proposed contact dynamics formulation, the challenges of handling stiff differential equations in the compliance based approaches will be avoided as no relaxation is imposed in the normal direction. Such a formulation, as well as its validity and applicability, will be addressed in this paper.

2. Dynamics formulation

Let us consider a mechanical system, the motion of which is parameterized by n generalized velocity components collected in $n \times 1$ array $\mathbf{v}$, and the configuration of the system is represented by n_c generalized coordinates given in $n_c \times 1$ array, $\mathbf{q}$. The time rate of change of $\mathbf{q}$ is related to $\mathbf{v}$ as

$$\dot{\mathbf{q}} = \mathbf{\Gamma}\mathbf{v}, \tag{1}$$