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Synthesis of the steepest rotation pitch curve design for noncircular gear

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ABSTRACT

A steepest rotation method for designing pitch curve with fixed boundaries of noncircular gear is proposed by resorting to calculus of variations. A general mathematical model is established to design the steepest rotation pitch curves with integral constraint for noncircular gear pair, including external and internal meshing gear pair. In particular, to achieve the design of pitch curve with epicyclic constraint for driven noncircular gear, the epicyclic constraint conditions are also established. In addition, the unified design algorithm of pitch curves of noncircular gear pair with this steepest rotation characteristic is given in this paper, the pitch curves of desired geometrical and transmission properties can be solved easily by using the proposed algorithm. Examples presented in this paper are implemented in MATLAB, and feasibility and validity of above algorithm are verified.

1. Introduction

Since noncircular gears are more compact and better balanced than linkages and cams, they have been already applied to mechanical equipments: Maltese mechanism, packaging machinery, printing machinery, textile machinery, gear shaper and flow meters [1–6]. Dooner D. B. and Seireg A. A. [7] proposed an interactive design approach of noncircular gears. Dooner D. B [8] used noncircular gears to eliminate unnecessary torque and speed fluctuations. Wu L. I. and Chang S. L. [9] obtained the tooth profiles of elliptical gears by combining the involutes generated from base curves with appropriate addendum and dedendum curves. Tong S. H. and Yang D. C. H. [10] obtained identical noncircular conjugate pitch curves with any number of lobes. Wei H. [11] developed an elliptic interpolation algorithm for elliptical gears on the basis of the principle of central angle division of an arc. Hector F. Q. R., Salvador C. F. and et al. [12] designed an approach for generating pitch curves of *N*-lobed noncircular gears on the basis of Bézier and B-spline nonparametric curves. Mundo D. [13] obtained the planetary gear train with specified variable gear ratio law by combing three noncircular gears. A general generation method of *N*-lobed elliptical gears from a basic ellipse was proposed in Ref. [14], and the synthesis of *N*-lobed or high-order elliptical gears and their rack by means of a conjugate shaper cutter with an involute tooth profile was obtained in Refs. [15–22].

Most of pitch curves for these noncircular gears in Refs. [1–22] were constructed by existing smoothed curves as pitch curves of noncircular gears. However, with the rapid development of numerical control machine tool and computer technology, some novel design methods of pitch curves for *N*-lobed noncircular gears have been proposed in recent studies. Yao W. X. [23] presented a pitch curve design method, which using plane regular *N*-curved polygon and spiral of Archimedes as the pitch curve for *N*-lobed noncircular gear. Another new design method of pitch curves with concave for noncircular bevel gears was proposed by Kan S. and Xia J. Q. [24].

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Although some novel pitch curves are put forward in succession, the high efficiency requirement of noncircular gear transmission still cannot be realized. In view of this, a kind of design method of steepest rotation pitch curves for noncircular gears is presented in this paper, the noncircular gears with steepest rotation pitch curves possess shortest transmission time at a given angular velocity. In addition, the pitch curves with this characteristic are more attractive than previous designs under the high efficient transmission requirement. Several typical design methods of steepest rotation pitch curves for noncircular gears based on calculus of variations are presented.

2. Design of steepest rotation pitch curve with fixed boundaries for noncircular gear

As depicted in Fig. 1, points A, B and O are two fixed endpoints and rotation center of plane curve C, respectively. Fixed coordinate system O-xy is attached to rotation center O, θ_a and θ_b are polar angles of points A and B, respectively. Assume that the polar equation of plane curve C is $r_{fb}(\theta)$, according to the principle of kinematics and differential, the differential $dt(\theta)$ of time function $t(\theta)$ that noncircular gear through plane curve $r_{fb}(\theta)$ can be expressed as:

$$dt(\theta) = \frac{ds(\theta)}{\omega r_{fb}} = \frac{\sqrt{{r_{fb}}^2 + {r'_{fb}}^2}}{\omega r_{fb}} d\theta \tag{1}$$

where $ds(\theta)$ and $r'_{fb}(\theta)$ are small section arc length and first-order derivative for plane curve $r_{fb}(\theta)$, respectively. Parameter ω is a given rotation angular velocity. Polar angle θ is measured counterclockwise from the positive direction x-axis.

Integrating both sides of Eq. (1) from θ_a to θ_b, we obtain:

$$T = \int_{\theta_a}^{\theta_b} dt(\theta) = \int_{\theta_a}^{\theta_b} \frac{\sqrt{{r_{fb}}^2 + {r'_{fb}}^2}}{\omega r_{fb}} d\theta \tag{2}$$

where T is rotation time that noncircular gear through plane curve $r_{fb}(\theta)$ at the given angular velocity ω from $\theta = \theta_a$ to $\theta = \theta_b$, and $0 \le \theta_a < \theta_b \le 2\pi$.

From the Eq. (2), we know that the value of time T may be different for different plane curve $r_{fb}(\theta)$, so the minimum T_{min} of rotation time can be expressed as:

$$\begin{cases} T_{\min} = \min T = \min \int_{\theta_a}^{\theta_b} F(\theta, r_{fb}, {r_{fb}}') d\theta \\ F(\theta, r_{fb}, {r_{fb}}') = (\omega r_{fb})^{-1} \sqrt{{r_{fb}}^2 + {r_{fb}}'^2} \end{cases} \tag{3}$$

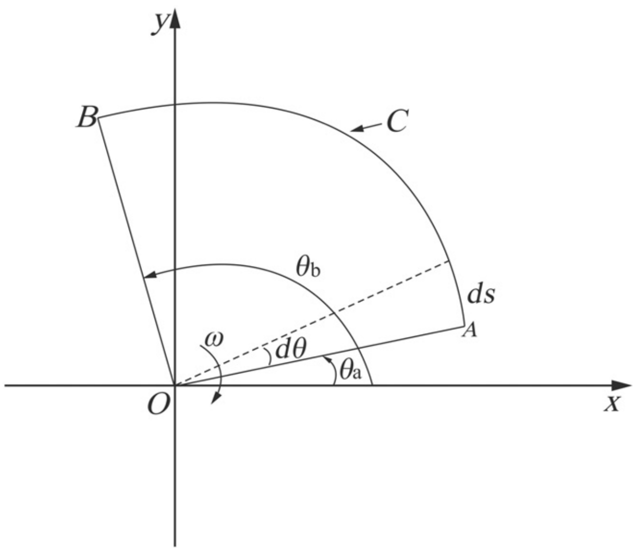

Fig. 1. Pitch curve of noncircular gear with fixed boundaries.