Mechanism and Machine Theory 102 (2016) 68–85

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Design of gravity compensators using the Stephenson and Watt mechanisms

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ARTICLE INFO

Article history:
Received 21 October 2014
Received in revised form 7 March 2016
Accepted 19 March 2016
Available online 16 April 2016

Keywords:
Associated linkage
Gravity compensation
Static balancing
Space mapping
Design equation
Deletion rule

ABSTRACT

This paper evaluates the design method for a gravity compensator using associated linkages. For conventional design methods the kinematics and potential energy of every mechanism should be computed to design a static balancer. For the proposed design method, however, no computations of kinematics and potential energy are necessary to obtain static balancers of various mechanisms, once a static balancer of an associated linkage has been designed. The deletion rules (i.e., transformation relations) are derived with the concept of the associated linkage. The Stephenson and Watt linkages are adopted as associated linkages that contain ternary links and multi-loops. Gravity compensators of the associated linkages are designed based on the space mapping method. When a ternary link becomes a slider, several unit gravity compensators are attached at the ternary link and can be merged into an equivalent gravity compensator. In consideration of the equivalent gravity compensator, a new deletion rule is introduced and incorporated in the set of the original deletion rules. Various gravity compensators are derived applying the new deletion rules to gravity compensators of associated linkages. Performances of the various gravity compensators are evaluated with simulations.

1. Introduction

Gravity compensators have been utilized in various fields including applications in rehabilitation [1–3], as serial [4–6] or parallel manipulators [7–9] and in face robots [10]. One-Degree Of Freedom (DOF) [11–14] and Multi-DOF gravity compensators have been proposed [15–17]. A gravity compensator for varying loads has also been recently proposed [18]. The energy method has been developed to determine the spring coefficient of a gravity compensator [19]. Streit and Gilmore [20] proposed a design method for an n-spring balancer for a one-link system with Two-DOF rotation. Agrawal [21] also developed a hybrid design. Deepak [22] proposed a static balancing method for a general n-DOF revolute and spherical jointed rigid-body linkage.

A design method has been proposed wherein springs are directly installed between links in the case of a planar manipulator, whereby a stiffness block matrix has been suggested [23,24]. Cho and Kang [25] proposed a design method based on space mapping that can simultaneously determine the number and locations (or kinematic constraints) of unit gravity compensators. Kim and Cho [26] introduced a design method based on the concept of the associated linkages wherein gravity compensators of derived mechanisms from the associated linkage can be designed from a gravity compensator of the associated linkage.

Type and dimensional syntheses are often used to design an appropriate mechanism. Type synthesis is applied to determine a kinematic structure of a mechanism. The concept of the associated linkage is introduced as a type synthesis method [27,28]. For the concept of the associated linkage a basic kinematic chain is selected and modified to obtain a desired mechanism. Consider a

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http://dx.doi.org/10.1016/j.mechmachtheory.2016.03.018

four-bar linkage as an example. When a revolute joint is replaced by a prismatic joint, a slider-crank mechanism, a swinging block linkage and a turning block linkage can be obtained in accordance with the location of a prismatic joint. When two revolute joints are replaced by prismatic joints, a cardanic motion linkage can be obtained. Thus, various mechanisms can be obtained by changing revolute joints into prismatic joints. A four-bar linkage becomes an associated linkage or a basic kinematic chain in this situation.

This paper evaluates the design method using associated linkages proposed in [26] for a mechanism that contains ternary links and multi-loops. A gravity compensator of a sliding mechanism can be designed with conventional design method (e.g., [19–25]). By applying a conventional design method, computations of kinematics and potential energy are necessary for every mechanism. In the proposed design method, however, kinematics and potential energy are computed once for an associated linkage. Gravity compensators of sliding mechanisms are obtained by transforming the gravity compensator of an associated linkage. Thus, no computations of kinematics and potential energy of sliding mechanisms are necessary. The previous research in [26] has been conducted on the four-bar linkage that possesses binary linkages and a single-loop. In this paper, more complicated mechanisms are considered.

Herein, the Stephenson and Watt linkages are adopted as associated linkages consisting of two ternary links and seven revolute joints, respectively. When a ternary link, quaternary link, quinary link or n-ary link is implemented in a mechanism, multi loops are observed and the offset of the mass center occurs inevitably. Consider a planar parallel mechanism as an example in that a ternary link is used as a moving platform. Three internal loops are observed and the offset of the mass center of the ternary link should be considered in designing a gravity compensator. Thus, the proposed method demonstrated with the Watt and Stephenson mechanisms can be extended to a general mechanism. It is also noted that the proposed method is demonstrated in the two-dimensional (planar) space. Five mechanisms can be derived from the Stephenson mechanism and four mechanisms can be derived from the Watt mechanism when a revolute joint turns into a prismatic joint [27,28].

This paper will propose a way to deal with ternary links. The offset of the center of mass inevitably occurs at the ternary link. When a ternary link becomes a slider, several unit gravity compensators are attached to it, because of the offset of the center of mass. In this case, such compensators can be merged into an equivalent gravity compensator. It is also shown in this paper that the potential energy of the equivalent gravity compensator is less than sum of the potential energies of the original gravity compensators. In this work, conversion rules are adjusted considering the offset of the center of mass at the ternary link. A new deletion rule is introduced for the equivalent gravity compensator and is incorporated into the original set of deletion rules. The number of possible joint spaces increases to describe all positions of the center of mass because of the multi-loop. Gravity compensators of the associated linkages are designed using the space mapping method [25]. Gravity compensators of derived mechanisms are designed by applying the new deletion rule to gravity compensators of the associated linkages and their performances are evaluated with simulations.

2. Design method with the associated linkage

2.1. Space mapping

The space mapping method is utilized to design a gravity compensator of an associated linkage and is briefly summarized in this section. The interested reader is referred to [25] for a more detailed explanation of this method. The design of a spring balancer is considered as a mapping between two spaces (i.e., the joint space for gravitational torques and the gravity compensator space for compensating torques). The mapping matrix expresses the mechanical connections of the one-DOF gravity compensators (or unit gravity compensators) with respect to the target mechanism.

Let the joint space be $\Theta=[\theta_1,\theta_2,\cdots,\theta_n]^T\in R^{n\times1}$, where θ_i and n denote the rotation angle of the i^{th} joint and the number of joints, respectively. One-DOF gravity compensators are utilized in this paper. The gravity compensator space is determined as $\Theta_g=[\theta_{g1},\theta_{g2},\cdots,\theta_{gm}]^T\in R^{m\times1}$, where θ_{gi} and m represent the rotation angle of the i^{th} unit gravity compensator and the number of the unit gravity compensators, respectively. Rotation angles of unit gravity compensators are determined passively by the pose of a mechanism (i.e., Θ). Thus, a mapping relation between the joint and gravity compensator spaces can be described as $\Theta_g=\mathbf{J}\Theta+\Phi$, where $\mathbf{J}\in R^{m\times n}$ and $\Phi\in R^{m\times1}$ denote the mapping matrix and the vector of constant phase angles, respectively.

The complete gravity compensation is achieved, when the total potential energy is invariant (i.e., $V(\Theta,\Theta_g)=V_m(\Theta)+V_k(\Theta_g)=$constant). Considering that $\Theta_g=\mathbf{J}\Theta+\Phi$, $V(\Theta,\Theta_g)$ becomes $V(\Theta)=V_m(\Theta)+V_k(\mathbf{J}\Theta+\Phi)$. Partial differentiation of $V(\Theta)$ with respect to θ_i results in the design equation of $\mathbf{f}(\Theta)\mathbf{V}_{m\_\max}$-$\mathbf{J}^T\mathbf{MK}=0$, where $\mathbf{f}(\Theta)\in R^{n\times n}$ and $\mathbf{V}_{m\_\max}=[V_{m1},V_{m2},\cdots,V_{mn}]^T\in R^{n\times1}$. In this case, V_{mj} indicates the maximum constant potential energy measured at a joint j, and $f_{ij}(\Theta)$ represents the ratio of the variance of V_{mj} by changing the pose of a given mechanism. Additionally, $\mathbf{M}=diag[\sin(\mathbf{J}_1\Theta+\phi_1),\sin(\mathbf{J}_2\Theta+\phi_2),\cdots,\sin(\mathbf{J}_m\Theta+\phi_m)]=diag[\sin(\theta_{g1}),\sin(\theta_{g2}),\cdots,\sin(\theta_{gm})]\in R^{m\times m}$ and $\mathbf{K}=[k_1b_1h_1,k_2b_2h_2,\cdots,k_mb_mh_m]^T\in R^{m\times1}$, where $diag[x_1,\cdots,x_n]$ denotes an $n\times n$ diagonal matrix and $\mathbf{J}^T$ represents the transpose of $\mathbf{J}$ (i.e., $\mathbf{J}^T\in R^{n\times m}$). The symbols h_j, b_j, and k_j represent parameters of the j^{th} gravity compensator.

The design equation (i.e., $\mathbf{f}(\Theta)\mathbf{V}_{m\_\max}$-$\mathbf{J}^T\mathbf{MK}=0$) shows the mapping relation between gravitational torques and balancers' torques. Let $\boldsymbol{\tau}_m\in R^{n\times1}$ and $\boldsymbol{\tau}_k\in R^{m\times1}$ be a torque vector by masses and a torque vector of gravity compensators, respectively. From the principal of virtual work, the infinitesimal work by the gravitational torque, $\delta\Theta^T\boldsymbol{\tau}_m$, is identical to the infinitesimal work by the compensating torque, $\delta\Theta_g^T\boldsymbol{\tau}_k$ (i.e., $\delta\Theta^T\boldsymbol{\tau}_m=\delta\Theta_g^T\boldsymbol{\tau}_k$). The relation between $\delta\Theta$ and $\delta\Theta_g$ is determined as $\delta\Theta_g=\mathbf{J}\delta\Theta$ from $\Theta_g=\mathbf{J}\Theta+\Phi$. Substitution of $\delta\Theta_g=\mathbf{J}\delta\Theta$ into $\delta\Theta^T\boldsymbol{\tau}_m=\delta\Theta_g^T\boldsymbol{\tau}_k$ yields $\delta\Theta^T\boldsymbol{\tau}_m=\delta\Theta^T\mathbf{J}^T\boldsymbol{\tau}_k$. Finally, we obtain $\boldsymbol{\tau}_m=\mathbf{J}^T\boldsymbol{\tau}_\mathbf{k}$. A comparison of $\boldsymbol{\tau}_m=\mathbf{J}^T\boldsymbol{\tau}_\mathbf{k}$ with the design equation yields $\mathbf{f}(\Theta)V_{m\_\max}=\boldsymbol{\tau}_m$ and $\mathbf{MK}=\boldsymbol{\tau}_k$.

The mapping matrix $\mathbf{J}$ can be determined by decomposing the eigenvalues of $\mathbf{f}(\Theta)$. Assume that the eigenvalues of $\mathbf{f}(\Theta)$ are $\boldsymbol{\lambda}=[\lambda_1,\lambda_2,\cdots,\lambda_n]^T\in R^{n\times1}$, and λ_i can be decomposed with some basis functions. Considering of $\mathbf{M}$ in the design equation, λ_i is