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Geometric construction and kinematic analysis of a 6R single-loop overconstrained spatial mechanism that has three pairs of revolute joints with intersecting joint axes

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ABSTRACT

This paper deals with the geometric construction and kinematic analysis of a single degree-of-freedom 6R overconstrained spatial mechanism (6R mechanism for short) that has three pairs of revolute (R) joints with intersecting joint axes. How to construct a 6R with three pairs of R joints with intersecting joint axes is presented first. The kinematic analysis of the 6R mechanism is then discussed. The analysis shows that the 6R usually has two solutions to the kinematic analysis for a given input. In two configurations in each circuit of the 6R mechanism, the axes of four R joints are coplanar, and the axes of the other two R joints are perpendicular to the plane defined by the above four R joints.

1. Introduction

The research on single degree-of-freedom (DOF) single-loop overconstrained mechanisms started more than one and half centuries ago. So far, a number of single-loop overconstrained spatial mechanisms [1–24] have been proposed. Meanwhile, different approaches, such as geometric methods [1,2,24], construction approaches [3,4,8,14,22], algebraic approaches [6,12,15–21,23] and numerical methods [7], have been developed for obtaining 6R mechanisms, which is composed of six R (revolute) joints. For a comprehensive list of 6R mechanisms identified before 2009, refer to Ref. [15]. A historic review of research on 6R mechanisms by Russian researchers can be found in Ref. [25]. In addition, an efficient method was proposed for calculating the DOF of an overconstrained mechanism in Ref. [26]. Despite the advances in the past decades, searching for 6R mechanisms is still an open issue.

Although the successful industrial applications of single-loop overconstrained mechanisms are not many so far, single-loop overconstrained mechanisms have been used in the development of deployable structures [14] and parallel mechanisms [27,28]. The application of 6R mechanisms in mobile robots [29] and other devices [25,30] is being explored.

Recently, several 6R mechanisms, which have two pairs of adjacent R joints with parallel joint axes and one pair of non-adjacent R joints with parallel joint axes, have been systematically obtained using a dual-quaternion based approach [19–21] or a construction approach [22,24]. Before the systematic type synthesis of this class of 6R mechanisms, only two such 6R mechanisms were proposed in the literature and both were unfortunately ignored by researchers working on overconstrained

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6R mechanisms for decades [11,18,19,22]. The first one was proposed by Mudrov in 1976, as pointed out in Ref. [25], and can be regarded as a special case of the double-Bennett hybrid 6R mechanism [4]. The second one was obtained during the investigation of novel steering techniques in Ref. [9]. Following the success of type synthesis of this class of 6R mechanisms, it is logic to obtain 6R mechanisms that have three pairs of R joints with intersecting joint axes, including two pairs of adjacent R joints with intersecting joint axes and one pair of non-adjacent R joints with intersecting joint axes. It is apparent that two 6R mechanisms that have three pairs of R joints with intersecting joint axes can be obtained from plane symmetric Bricard mechanism and line symmetric Bricard mechanism by imposing certain geometric constraints, like in the generation of 6R mechanisms that have three pairs of R joints with parallel joint axes [22]. Whether there are other classes of 6R mechanisms that have three pairs of R joints with intersecting joint axes has not been investigated.

Inspired by the geometric construction of Bricard 6R mechanisms [1,2], especially the trihedral Bricard mechanism and the Type III Bricard's flexible Octahedron, this paper aims at revealing a 6R mechanism that has three pairs of R joints with intersecting joint axes using a geometric construction approach.

This paper is organized as follows. The geometric construction of a 6R that has three pairs of R joints with intersecting joint axes will be given in Section 2. The kinematic analysis of the 6R will be presented in Section 3. In Section 4, the characteristics of the 6R mechanism will be revealed. Finally, conclusions will be drawn.

For simplicity reasons, $\sin\theta_i$ and $\cos\theta_i$ are denoted by $S\theta_i$ and $C\theta_i$, respectively.

2. Description of a 6R that has three pairs of R joints with intersecting joint axes

In this section, the geometric construction of a 6R is presented first. The D–H (Denavit–Hartenberg) link parameters of the 6R mechanism are then given.

2.1. Geometric construction of a 6R mechanism that has three pairs of R joints with intersecting joint axes

A 6R mechanism that has three pairs of R joints with intersecting joint axes can be constructed as follows (Fig. 1):

Step 1: Draw six lines for placing R joints. At first, draw a kite ACBD where $|AC|=|BC|$ and $|AD|=|BD|$ as well as two circles of radius r ($r \leq |AC|=|BC|$ and $r \leq |AD|=|BD|$) with their centers at A and B respectively. Then draw:

- Lines AA' and BB' that are perpendicular to the kite ACBD.
- Lines CA_1 and CB_1 that are tangent to circles A and B respectively at points A_1 and B_1 such that $\angle ACA_1=\angle BCB_1$.
- Lines DA_2 and DB_2 that are tangent to circles A and B respectively at points A_2 and B_2 such that $\angle ADA_2=\angle BDB_2$.

Lines AA', BB', CA_1, CB_1, DA_2 and DB_2 are the six lines required for placing six R joints.

Step 2: Construct a 6R mechanism using the six lines obtained in Step 1.
Place six R joints 1, 2, $\cdots$ 6 along lines CA_1, AA', DA_2, DB_2, BB', and CB_1 respectively and connect them in the sequence of 1-2-3-4-5-6-1. One then obtains a 6R mechanism that has three pairs of R joints with intersecting joint axes 1-2-3-4-5-6-1. Here two adjacent R joints are connected using a circular link if the two joint axes intersect with each other or a straight link if the the two joint axes do not intersect.

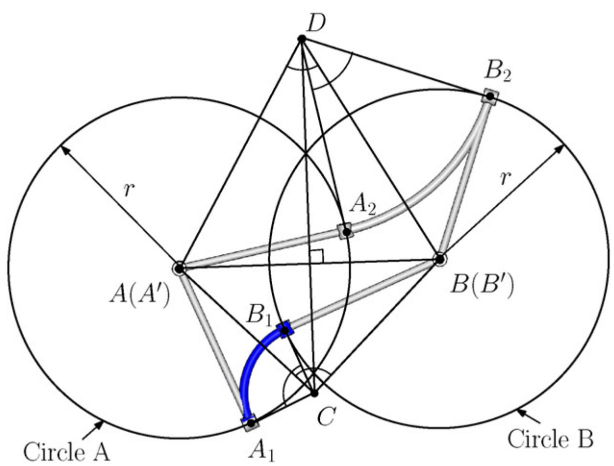

Fig. 1. Construction of a 6R mechanism.