Mechanism and Machine Theory 98 (2016) 21–35

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Dynamic trajectory planning of a 3-DOF under-constrained cable-driven parallel robot

Nan Zhang, Weiwei Shang *

Department of Automation, University of Science and Technology of China, Hefei 230027, PR China

ARTICLE INFO

Article history:
Received 25 February 2015
Received in revised form 11 November 2015
Accepted 14 November 2015
Available online xxxx

Keywords:
Cable-driven robots
Static equilibrium workspace
Trajectory planning
Under-constrained robots

ABSTRACT

In this paper, the trajectory planning of a spatial 3-dof under-constrained cable driven parallel robot is studied. We propose a geometrical approach to plan trajectories that extend beyond the static equilibrium workspace (SEW) of the mechanism. First, conditions for cable tensions to be positive along a straight line path are given geometrically. Then, a point-to-point trajectory which satisfies the cable tension constraints is designed in s–$\ddot{s}$ plane, where s denotes the path coordinate. The parameter of the trajectory can be selected by making a trade-off between the safety of cable tensions and the duration of the trajectory. To avoid failing to connect prescribed points, a new workspace is defined such that any target points in the workspace can be reached in sequence. Periodic trajectories including oscillations along a straight line and uniform circular motion in a horizontal plane are designed in a similar way. According to the geometric properties of the cable tension constraints, the range of the periodic trajectory parameters can be readily determined. The proposed approach yields analytical results and can ensure positive and continuous tensions in cables.

1. Introduction

Cable driven parallel robots (CDPRs) are mechanisms which utilize cables instead of rigid links to actuate an end-effector. CDPRs strengthen classic advantages of parallel robots relative to serial robots while providing a number of new desirable characteristics, including: 1) high payload-to-weight ratios; 2) high speed manipulation; 3) energy efficient; 4) easy to assemble, disassemble, reconfigure and transport; and 5) potentially large workspace. Due to these features, CDPRs are well suited for many applications such as automated construction systems [1], haptic interfaces [2], positioning telescopes [3], camera systems for stadiums [4], rescue devices [5], and machines for rehabilitation [6].

However, a major drawback of CDPRs is that cables may become slack while the end-effector is moving. In order to make the CDPRs operate properly, positive tensions in cables must be guaranteed. For the fully-constrained CDPRs [7], the controllable workspace [8] (also known as the force-closure workspace [9] or the wrench-closure workspace [10]) can be used to avoid cable slackness during the robot motion [11]. However, for under-constrained CDPRs which usually use gravity to maintain cables in tension at rest (zero velocity and zero acceleration), the controllable workspace does not exist. So, the positive cable tension constraints should be carefully considered for any motion of the under-constrained CDPRs, and generally, these constraints are complex, which makes the trajectory planning of under-constrained CDPRs a challenging problem.

* Corresponding author. Tel.: +86 55163601332; fax: +86 55163603244.
E-mail address: wwshang@ustc.edu.cn (W. Shang).

http://dx.doi.org/10.1016/j.mechmachtheory.2015.11.007
0094-114X/

Several contributions proposed in literature have dealt with this problem. In [12], the time-optimal trajectory planning was studied on a cable-based robot for a given geometrical path. The authors first reduced the multi-dimensional trajectory planning problem down to two: position and velocity (path coordinate), and expressed the cable tension constraints in terms of the path coordinate. Then the phase plane technique given in [13–15] was used to find a trajectory which satisfies the constraints and its corresponding motion time is minimum. However, the method cannot guarantee the continuity of tensions in cables, and the computation time for a time optimal trajectory is much larger than the motion time. In [16], a smooth trajectory planning algorithm based on quintic polynomials was adopted for a hybrid planar 2-dof under-constrained CDPR. The technique can guarantee the continuity of cable tensions, but it cannot be used to plan trajectories that extend outside the static equilibrium workspace (SEW [17]) of the robot. In [18], the authors took into consideration a planar 2-dof under-constrained CDPR. Based on the dynamic model of the robot, algebraic inequalities were obtained that represent the constraints on cable tensions. Then, by choosing a special frequency, conditions were obtained for the feasibility of families of periodic trajectories. The technique was extended to a spatial 3-dof under-constrained CDPR in [19], and further, the accuracy of the trajectories was determined experimentally in [20]. These trajectories can go beyond the SEW of the mechanism, thereby opening novel possibilities for the robot. However, the periodic trajectories designed in these papers are too special and are difficult to be extended to the general case. Moreover, using natural frequency to reduce the complexity of the algebraic inequalities makes the periodic trajectories have no parameters to be adjusted by the user. In [21], two trajectory-planning techniques for the point-to-point motion of a planar 2-dof under-constrained CDPR were introduced. The authors first designed polynomial or trigonometric functions to connect the target points, then used cable tension constraint inequalities to verify the feasibility of the planned trajectories. The techniques can be used to plan point-to-point trajectories extending beyond the SEW, but may fail to connect prescribed points.

In this paper, the dynamic trajectory planning of a spatial 3-dof under-constrained CDPR is addressed. Instead of using the algebraic techniques given in [18–21], a geometric approach is proposed to plan trajectories that extend beyond the SEW of the robot. First, the positive cable tension constraints are expressed geometrically in s–$\ddot{s}$ plane. Then, a parametric point-to-point trajectory which satisfies the cable tension constraints is given. The range of the parameter of the trajectory can be computed analytically, and the relationships among the parameter, the duration of the trajectory and the safety of cable tensions (i.e., the condition of being protected from cable slack) are obtained. In order to avoid failing to connect prescribed points, a new workspace called the reachable workspace is defined. The reachable workspace can be obtained analytically if the starting point of the point-to-point trajectory is given and its volume is four times as large as the volume of the SEW. Using the proposed trajectory planning approach, any prescribed points in the reachable workspace can be connected in sequence and continuity up to the acceleration level can be ensured. The geometric approach can also be used to design periodic trajectories, e.g., oscillations along a straight line and uniform circular motion in a horizontal plane. For oscillations along a straight line, the midpoint ($\boldsymbol{p}_0$) of oscillation should lie inside the SEW and the maximum amplitude ($A_{p\ \max}$) of oscillation can be determined if $\boldsymbol{p}_0$ and the direction of the line are given. For any given amplitude (A_p) satisfying $A_p \in (0, A_{p\ \max})$, the range of oscillation frequency can be computed analytically. For uniform horizontal circular motion, if the center ($\boldsymbol{o}$) of the circle lies in the SEW, theoretically, the radius (R) of the circle can be selected arbitrarily and the range of frequency can also be determined analytically if $\boldsymbol{o}$ and R are given.

This paper is arranged as follows. Section 2 presents the kinematic and dynamic models. Section 3 displays the positive cable tension constraints along a straight line path. In Section 4, a geometry based trajectory planning approach is introduced. Methods of planning periodic trajectories are demonstrated in Section 5. In Section 6, simulation results are discussed. Finally, concluding remarks are given in Section 7.

2. Kinematic and dynamic modeling

The structural diagram of the studied spatial 3-dof CSPR is shown in Fig. 1. The robot consists of three cables and three actuated spools which are mounted on a fixed structure. $OXYZ$ is the base coordinate frame and $\boldsymbol{A}_1$, $\boldsymbol{A}_1$, and $\boldsymbol{A}_3$ are the cable exit points which lie on the horizontal plane OXY and are not collinear. The three cables come out from the cable exit points and connect to an end-effector which is considered as a point mass. By controlling the extension of the cables, the position of the end-effector can be controlled. Here, the position of the end-effector is denoted by $\boldsymbol{p}$, and the cable lengths are denoted by l_i, $i = 1,2,3$. Then, the inverse kinematic equations can be written as

$$l_i = ||\boldsymbol{p} - \boldsymbol{A}_i||, \quad i = 1, 2, 3. \tag{1}$$

The cable tension vectors are denoted by $\boldsymbol{t}_i = t_i\boldsymbol{e}_i$, $i = 1,2,3$, where t_i is the value of the i-th cable tension and $\boldsymbol{e}_i = (\boldsymbol{A}_i - \boldsymbol{p})/l_i$ represents the direction of the i-th cable. According to Newton's second law of motion, the dynamic model of the robot is written as

$$\boldsymbol{JT} = m(\ddot{\boldsymbol{p}} - \boldsymbol{g}), \tag{2}$$