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An improved analytical method for mesh stiffness calculation of spur gears with tip relief

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ABSTRACT

Due to the effects of gear flexibility, the extended tooth contact (ETC) can appear, which is the phenomenon that the incoming tooth pair gets into contact ahead of the theoretical start of contact and the outgoing tooth pair is out of contact later than the theoretical end of contact. A large calculation error for the time-varying mesh stiffness (TVMS) calculation can be caused if the effects of ETC are ignored, especially under the larger torques. In this paper, an improved analytical method (IAM) suitable for gear pairs with tip relief is established to determine time-varying mesh stiffness (TVMS), where the effects of ETC, nonlinear contact stiffness, revised fillet-foundation stiffness, and tooth profile modification are considered. Based on the improved analytical model, TVMS under different torques, lengths, and amounts of profile modification is compared with that obtained from analytical finite element approach [29] and from FE method. The results show that TVMS obtained from the IAM agrees well with that from FE method and from analytical FE approach [29], and the computational efficiency of the IAM is also much higher than that of FE method.

1. Introduction

Time-varying mesh stiffness (TVMS) is a periodic internal excitation of gear pairs, which is very important for the vibration analysis of gear systems. Many researchers have devoted their efforts to investigating the calculation methods of TVMS. These calculation methods can be classified into four main groups: finite element (FE) method [1–8], analytical method [9–27], analytical FE approach [28–32], and experimental method [33–35]. There is no doubt that the FE method is the most powerful method to calculate TVMS because both the effects of tooth errors and flexibility of gear can easily be considered in it. Wang and Howard [2] calculated the torsional stiffness of an involute spur gear pair using FE method. Without considering the effects of extended tooth contact (ETC), Ma et al. [6] established an FE model for a gear pair with tip relief and further took the effects of ETC into account [7]. Analytical FE approach uses a linear FE method to calculate tooth deflections and adopts the Hertzian contact theory to obtain the local contact deformation, which can simulate the effects of ETC. Based on an analytical FE approach, the loaded static transmission error (LSTE), load ratio, and TVMS for spur gear pairs are studied by Fernandez del Rincon et al. [28,29], and mesh stiffness for helical gears is determined by Hedlund and Lehtovaara [30] and Chang et al. [31]. Using the analytical FE method, Pedersen and Jørgensen [32] pointed out that the boundary condition through the gear rim size and the size of the contact have a major influence on the stiffness.

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Although the TVMS obtained from the FE method is more accurate, the method is time-consuming. Hence, many analytical models are also developed to calculate the TVMS. Attia [9] deduced deflections and stiffness by calculating component deflections of the teeth and those produced by different deformations in the tooth-root fixations, the gear body, and the loaded neighboring teeth. Taking the Hertzian contact energy, bending energy and axial compressive energy into account, Yang and Lin [10] calculated the TVMS based on a potential energy method. On the basis of the model [10], Tian [11] and Wu et al. [12] further added the effects of the shear mesh stiffness. Simplifying the gear body as a cantilever beam in a half plane, Zhou et al. [13] developed an analytical model to calculate the TVMS considering the effects of the fillet-foundation deflection. In order to reduce the error of fillet-foundation stiffness caused by assuming the gear body as a cantilever beam in a half plane, Sainsot et al. [14] proposed an improved fillet-foundation deflection calculation method. Based on the results in Refs. [9,15], Chaari et al. [16] developed an analytical model for TVMS calculation, which considers Hertzian contact stiffness, tooth bending stiffness, and fillet-foundation stiffness. Chaari's model [16] was further extended by Chen and Shao [17] by introducing the shear stiffness and axial compressive stiffness. Many papers in the literature assume that the tooth is clamped at base circle; however, the tooth is actually fixed at root circle. In order to overcome this defect, Wan et al. [18] and Liang et al. [19] developed modified methods to obtain the TVMS accurately. In their methods, different formulas of bending potential energy are corrected when the number of gear teeth is less or more than 42. Ma et al. [20,21] also developed an analytical model for TVMS calculation, in which the formula of bending potential energy is not related to the number of gear teeth. For the gear with crack or tooth profile modification, some models are also proposed [22,23]. Taking the effects of the gear tooth errors into account, Chen and Shao [22] proposed a general analytical model to analyze the effects of tooth profile modification on TVMS, LSTE, and load-sharing factor. Gu et al. [24] proposed approximate formulae to calculate the time-varying mesh stiffness for ideal solid spur and helical gears.

In many analytical models [16–18,20–23], which consider the fillet-foundation stiffness (called traditional analytical method (TAM) in this paper), the total mesh stiffness in multi-tooth engagement is calculated by direct summation of TVMS of tooth pairs in contact. In fact, the contacting teeth are all connected to the same gear body when the multiple teeth are in mesh. Hence, the TAM overestimates the fillet-foundation stiffness and produces higher mesh stiffness than the real value during the meshing of multiple pairs of teeth. Moreover, in these models, the effects of gear flexibility are ignored when these models determine which pairs of teeth are in contact. The gear flexibility can cause the incoming tooth pair to enter contact earlier than the theoretical start of contact, and the outgoing tooth pair to leave contact later than the theoretical end of contact [25–27], namely, the phenomenon of ETC. The mesh stiffness, loaded static transmission error, and contact ratio considering the effects of ETC are different from those obtained from TAM [25–27].

FE methods [1–8] and analytical FE approaches [28–32] can consider the effects of ETC because both methods can easily calculate the deformations of gear teeth and gear bodies and can conveniently determine the number of the meshing tooth pairs. For the analytical methods, more and more attention is also being paid on the effects of ETC. Considering the effects of ETC, Tse and Lin [25] investigated the effects of separation distance and deflection of gear teeth on the static transmission error using an analytical method. A further investigation indicated that neglecting the extension of the contact zone can result in underestimating resonant speeds and overestimating the dynamic load, especially for heavily loaded gears [26]. Assuming that the rigid tooth and rigid wheel are connected to each other using elastic elements, a rigid-elastic modeling method of meshing gear pairs is developed in [36]. Considering the effects of ETC, an improved analytical model for TVMS calculation of cracked spur gears is proposed by Ma et al. [37]. Duverger et al. [38] developed an original torsional model to simulate the dynamic behavior of spur gears in cases of premature engagement and delayed recess.

Many of the analytical methods mentioned above overestimate the mesh stiffness during double-tooth engagement because they repeatedly consider the stiffness of gear body using the direct summation of the mesh stiffness of tooth pairs and ignore the fact that two or more meshing tooth pairs share the one gear body [1,37]. Moreover, the effects of tooth flexibility [25,26,37] are usually ignored for analytical models when these models are used to determine which pairs of teeth are in contact. In this study, an improved analytical model for the calculation of TVMS of gear pairs with tooth profile modification (TPM) is developed by correcting the two shortcomings mentioned above. The proposed method is also verified by comparing the results obtained from published references [22,29] and from FE method. In addition, the effects of the amount and length of profile modification on the TVMS are also evaluated. The detailed structure of this paper is shown in Fig. 1.

2. Time-varying mesh stiffness calculation

A general expression of TVMS of spur gear pairs is developed in Section 2.1 by adopting the revised fillet-foundation stiffness and nonlinear contact stiffness. This TVMS calculation model is further improved in Section 2.2 by considering the effects of ETC and gear tooth errors such as TPM.

2.1. TVMS calculation model adopting revised fillet-foundation stiffness

The gear tooth is generally modeled as a nonuniform cantilever beam and the mesh stiffness of single-tooth pair is [20,21]

$$k^i = \frac{1}{\frac{1}{k_{hi}} + \frac{1}{k_{t1}} + \frac{1}{k_{f1}} + \frac{1}{k_{t2}} + \frac{1}{k_{f2}}} \tag{1}$$