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Optimal design under uncertainty of bearing arrangements

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ABSTRACT

This study presents a methodology for an optimal design under uncertainty of a real bearing arrangement. The aim is to find the optimal bench bearing arrangement preload/clearance that maximizes the life of the bearing arrangement considering several uncertain geometric and operating parameters. The paper also presents a new systemic method used to determine the bearing loads required for the life calculations of the bearing arrangements. This method is suitable to be used within an Evolutionary Algorithm due to the fact that it is fast and reliable. The optimal design of the considered bearing arrangement—from the maximum bearing life standpoint—is achieved using a new and efficient stochastic optimization tool obtained by combining the features of Cuckoo Search algorithm and the ones of the Knowledge Gradient policy. Also, a specific approach for optimization under uncertainty using three stages is proposed in this paper. The obtained results are validated by Monte Carlo Sampling.

1. Introduction

Production of large amounts of bearing arrangements destined to work in different applications requires a special management than unique assemblies with the mating parts already measured and the working temperature very precisely estimated or known. For the latter any company can offer reliable information regarding the life of the bearing arrangement, but for the former it is almost impossible to make an accurate prediction without a powerful tool.

The bearing arrangement considered for optimization in this study is a particular one from the first category. Considering that the purpose is to find the optimal design that maximizes the bearing arrangement life based on the value of the bench bearing arrangement preload/clearance an optimization tool and one or more objective functions (based on reliable but fast models) and constraints have to be used in the early phase of the design. Even if it might seem as a simple problem at first glance, it is actually quite complicated because the lifetime calculation requires the real compressive forces that act on the rolling elements of the bearings to be known. These forces depend on the loads transmitted from the shaft to the bearings and therefore, a simple, reliable, but fast model has to be used for bearing load calculation within the optimization tool.

The Finite Element Analysis (FEA) is not a viable option due to the extremely long necessary running time. Even conducting an optimization with an accelerated Evolutionary Algorithm (EA) the order of magnitude of the number of the objective function calls is of hundreds of thousands and therefore, other model than FEA has to be used. Furthermore, the early phase of the design is governed by uncertainty. Even though the tolerances are known, the exact values of the interference between the shaft and the bearing inner ring bore and of the clearance between the housing bore and the bearing outer ring are unknown. Also, the working temperature cannot be rigorously identified, it can only be estimated. Considering the abovementioned aspects, the need for an efficient optimization tool, a fast model to substitute FEA, and a specific approach to deal with uncertainty is indisputable.

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Optimizations under uncertainty have been conducted in other fields or subfields such as portfolio management [1], option pricing [2], medicine [3], mining sector [4], chemistry [5], logistics [6], and engineering [7]. In another train of thoughts many scientific papers deal with the optimization of different machine elements, but there is a small number of available papers focused on the optimization of the rolling bearings. Charabotry [8] describes a mono-objective optimization of a ball bearing having five design variables using genetic algorithms. The aim was the maximization of the fatigue life. Rao and Tiwari [9] presented an optimal design methodology of ball bearings based on the maximization of the lifetime. The mono-objective optimization was also conducted using genetic algorithms. A multi-objective optimization with non-conflicting objectives was conducted using NSGA II (non-dominated sorting based genetic algorithm) [10]. Another multi-objective optimization with two objective functions (the basic dynamic radial load rating of the bearing and the minimum thickness of the elastohydrodynamic film between rollers and raceways) was performed using a multi-objective EA in order to obtain the optimal design of a single-row cylindrical roller bearing [11]. However, none of the papers presents an optimization under uncertainty, despite the fact that a high degree of uncertainty could be associated with the design and manufacturing of rolling bearing arrangements.

2. Research background

2.1. Bearing load models

After Sjoväll's pioneering work [12], probably the first general equations for the elastic equilibrium of a ball bearing in three of the five possible degrees of freedom were given by Jones in 1946 [13]. Several years later he has brilliantly completed his work [14] and a general model was issued, whereby the elastic compliances of a system of any number of ball and radial roller bearings under any system of loads can be determined. The system approach signifies that the entire assemblage of bearings, shaft, and supporting structure was looked at as a single, elastic system. The solution defines the elastic compliance of a point on the shaft with respect to the supporting structure in five degrees of freedom. Considering also the centrifugal forces and gyroscopic moments acting on the rolling elements, the internal load distribution is determined for all of the bearings in the system. Finally, bearing lives are evaluated by summation of the fatigue effects of the passages of the rolling elements over precisely determined paths in each bearing raceway.

In a two-part work, De Mul et al. constructed a general mathematical model for the calculation of the equilibrium and associated load distribution in both ball [15] and roller bearings [16]. The bearings may be loaded—with known loads and moments—and displaced in five degrees of freedom. The analysis is made with and without taking into account the centrifugal forces acting on the rolling elements, whilst the internal friction is neglected.

In order to derive a bearing stiffness model for vibration transmission analysis Lim and Singh [17] had to establish the relationships between the known bearing loads and moments transmitted through the rolling element bearing and the bearing displacements in 5 DOF. The reader could find more details in the authors' previous work [18]. In 2012, Gunduz [19] continued this work and developed the formulation of the stiffness matrix for a double-row angular ball bearing.

Houpert [20] proposed a so-called *uniform analytical approach* for ball and roller bearings which provides simple analytical equations to calculate the bearing loading (three loads and two tilting moments) based on the bearing raceway relative displacements (5 DOF). The interesting component of this approach is the manner of introducing the so-called *equivalent displacements* and expressing the rolling element-race load as a function of them. Moreover, the three components of the load and the two components of the moment on the inner raceway are calculated by integration, not by discrete summation. In 2014, Houpert strongly enhanced his model [21], especially for roller bearings.

Hernot et al. [22] presented two stiffness matrices of angular contact ball bearings. Using the two leading Sjoväll's load-distribution integrals J_a and J_r, the summation of ball-raceway loads was replaced by an integration and, in this way, the matrix connected to the conventional model in 2 DOF is first introduced. Using the constructed model, a study of a two bearing-shaft assembly where shaft deformations are ignored, was carried out. But, by taking preload into account it was clearly demonstrated how the influence of the preload on the assembly rigidity and bearing fatigue life may be analyzed. Conclusively, the matrix formulation of the 5 DOF model, connected with the Houpert's early model [20], is presented.

In two successive papers, Liao and Lin first established [23] and then developed [24] a tree-dimensional expression for the elastic deformation of bearing balls in terms of the geometry of the contact surface and the inner and outer raceway positions. Bai and Xu [25] reported a dynamic model of ball bearings used to study the dynamic properties of a rotor system supported by ball bearings under the effect of both internal clearance and raceway waviness. The proposed model includes centrifugal forces and gyroscopic moments.

Relative to our preeminent topic, in the fundamental two-volume monograph, Harris and Kotzalas presented either the Sjoväll's model of load distribution within ball and roller bearings under given external radial and axial load [26], or, partially [27], the Jones' already mentioned work.

Recent works are focused on obtaining the bearing stiffness matrix by extending the Jones' approach as Noel et al. [28] or by using FEM as Guo and Parker [29]. In both approaches, the external bearing loading has to be known. Relevant results were issued in the latter quoted paper, regarding the bearing radial and axial stiffness, respectively the obtained radial/axial stiffness–load relationship for both radial cylindrical roller and ball bearings being significantly different from those predicted by Gargiulo's well-known equations [30] (perhaps because these old equations do not take into account the elasticity of bearing rings).