Mechanism and Machine Theory 98 (2016) 199–215

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Dynamics of belt-pulley-shaft systems

Sanjib Chowdhury *, Rama K. Yedavalli [1]

Department of Mechanical and Aerospace Engineering, The Ohio State University, 201 West 19th Ave., Columbus, OH 43210, USA

ARTICLE INFO

Article history:
Received 22 February 2014
Received in revised form 1 November 2015
Accepted 23 November 2015
Available online xxxx

Keywords:
Linear dynamics
Gyroscopic rotating shaft
Belt-pulley-shaft systems
Extended operator
Galerkin's method

ABSTRACT

An analytical model of a pair of pulleys mounted on compliant spinning parallel shafts is developed. These form parts of a belt drive system and are connected by a single belt span, which is modeled as a combination of longitudinal and torsional springs. The pulleys are modeled as rigid cylindrical disks and slippage at the belt-pulley interfaces is ignored. The shafts are modeled as continuous beams having torsional flexibilities as well. The rotation speed is high such that the gyroscopic effect is non-negligible. Natural frequency sensitivity to rotation speed (Campbell diagram) and response due to a sinusoidal excitation force on a shaft is obtained. Modal analysis after reducing the discretized system to a first-order form is used for the response calculation. The study shows that the gyroscopic effect is present even for short lengths of the shafts. Splitting of the natural frequencies as well as the mode shapes is observed at low rotation speeds and critical speeds are observed at high rotation speeds. For coupled frequencies, the response of the system is found to be approximated by modal superposition of a smaller number of modes. The peak responses are explained using the Campbell diagram along with the modal energy plots.

1. Nomenclature

Subscripts 1 and 2 denote the quantities belonging to the first and the second shafts, respectively. An overbar indicates a dimensional quantity; otherwise, it is non-dimensional.

t	Time
Ω_1, Ω_2	Rotational speeds of the shafts
$\overline{L}_1, \overline{L}_2$	Lengths of the shafts
$\overline{\rho}_{s1}, \overline{\rho}_{s2}$	Shaft masses per unit length
u_1, u_2	Flexural deflections of the shafts in the uncoupled plane
u_{c1}, u_{c2}	Flexural deflections of the centers of mass of the pulleys in the uncoupled plane
v_1, v_2	Flexural deflections of the shafts in the coupled plane
v_{c1}, v_{c2}	Flexural deflections of the centers of mass of the pulleys in the coupled plane
θ_1, θ_2	Torsional deflections of the shafts about their respective axes
R_1, R_2	Radii of the pulleys
h_1, h_2	Thickness of the pulleys
h_{c1}, h_{c2}	Distances of the centers of mass of the pulleys from the nearest shaft ends
m_1, m_2	Masses of the pulleys
I_{cy}, I_{cz}	Mass moments of inertia of a pulley with respect to the y and z axes
I_m	Mass moment of inertia of a pulley with respect to the x axis
I_{sy}, I_{sz}	Area moments of inertia of a shaft cross-section with respect to the y and z axes

* Corresponding author.
E-mail addresses: chowdhury.22@osu.edu (S. Chowdhury), yedavalli.1@osu.edu (R.K. Yedavalli).
[1] Tel.: +1 614 292 3983; fax: +1 614 292 3163.

http://dx.doi.org/10.1016/j.mechmachtheory.2015.11.011
0094-114X/

J_1, J_2	Polar moments of inertia of shaft cross-sections
k_b	Longitudinal belt stiffness
K_{Tz}	Torsional belt stiffness about the direction of the belt travel
K_1, K_2	Torsional rigidities of the shafts
K_3, K_4	Torsional moments of inertia of the shafts
K_5	Mass per unit length of the second shaft
K_6	Flexural rigidity of the first shaft in the coupled plan
K_7, K_8	Flexural rigidities of the second shaft in the uncoupled and the coupled planes, respectively
λ_i	The i^{th} mode
ω_i	Natural frequency of λ_i

2. Introduction

Belt drive systems belong to the broad context of axially moving media including but not limited to power transmission chains, band saws in the wood and metal industries, magnetic tapes, and even pipes conveying flowing fluids. Vibration of the axially moving elements in these systems can be critical for the operation, e.g., large transverse vibration of the band results in poor cutting accuracy and surface quality in the wood industry. High noise level propagating from the supporting structures or directly from the vibrating band itself can be an issue from NVH standpoint. Alspaugh [1] investigated the torsional vibration of a thin rectangular strip translating at a constant speed in the longitudinal direction. Torsional buckling of the strip was predicted by the frequency-load diagram. Mote [2] analyzed small vibrations of a moving band saw theoretically and showed that the flexural natural frequencies always decrease continuously from a maximum at zero speed to zero value with increasing velocity. The rate of decrease depends on the relative motion between the band and the pulley axes. Fixed pulley axes results in constant band tension and a rapid decrease in the natural frequency with speed. On the other hand, if the band is allowed to extend under dynamic load, the natural frequency decreases less rapidly. Hence, pulleys mounted on flexible shafts will have the flexibility of the shafts influencing band tension fluctuation.

Belt drive systems are used as an effective means for transmission of power and are found in various applications such as conveyors, machine tools, stationary or mobile powered rotating equipment, etc. The advantages over gear and chain drive systems are low cost, light weight, quiet operation, easy maintenance, and flexible locations of the driver and driven shafts. A belt drive system consists of a few components such as belts, pulleys, shafts carrying the pulleys, etc. Torsional vibration of the shaft of a belt drive transmission in a precision machine tool was investigated by Mashinostroeniya [3]. The belt was represented as a weightless link possessing elastic and damping properties. The equations of motion consisted of the torsional equations of motion of the two pulleys. The parameters were determined from the amplitude-phase frequency characteristics (APFC) plots obtained in the test rig. Experiments on precision boring machines showed that the amplitude of the tool vibration was caused by disturbances derived from the drive and amounted to 20%–30% of the total relative vibration amplitude between the tool and the workpiece.

Until 1979, V-belts were exclusively used in automotive accessory drive systems. However, the lifespan of a V-belt is small and maintaining proper tension throughout the belt life is difficult. Nowadays, serpentine belt drives are widely used in automobiles and heavy vehicles for driving accessories such as alternators, air conditioners, water pumps, etc., by the engine power delivered from the crankshaft (Fig. 1). These are advantageous over the conventional V-belt drives with respect to compactness, longevity, simplified assembly, and proper maintenance of the belt tension. Despite these advantages, serpentine belt drives suffer from noise and belt tension fluctuation, which have their roots in the system vibration.

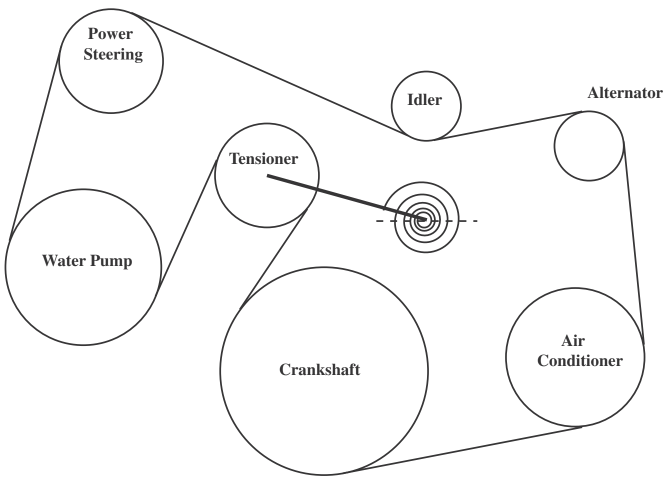

Fig. 1. Schematic of a serpentine belt driving an automotive engine accessories.