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Calculation of tooth bending strength and surface durability of internal spur gear drives

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ABSTRACT

In this paper the determinant load conditions for bending and pitting calculations, for involute, internal spur gears have been established. Calculations are based on a new model of load distribution, which takes into account the changing rigidity of the pair of teeth along the path of contact. This model has been obtained from the minimum elastic potential criterion, resulting in non-uniform load sharing among spur tooth pairs in simultaneous contact. As a result, the theoretical values of both bending and pitting nominal stresses are given, which may be suitable for standardization purposes.

1. Introduction

Gear calculation Standards [1–3] use simple equations given by the linear theory of elasticity, as Navier's equation, and the Hertzian contact model to evaluate the load capacity of spur and helical involute gear drives. But these equations are not in good agreement with experimental results if the load is assumed to be uniformly distributed along the line of contact, and several correction factors for load distribution are required [1,4]. In fact, the load distribution depends on the meshing stiffness of the pair of teeth, which is different at any contact point, so the load per unit of length is also different at any point of the line of contact, as well as at any point of the path of contact.

Some studies on the load distribution along the line of contact of tooth gears can be found in technical literature. Hayashi [5] obtained the load distribution as the numerical solution of the equation of Fredholm. Winter and Placzek [6] developed a model for load distribution taking into account profile modifications. Zhang and Fang [7] considered also generation errors and surface deformations. Boerner [8] used the coupling stiffness method to compute the load distribution and found that the curves of deformation were similar to those obtained from the bending solicitations. Ajmi and Velex [9] proposed a general approach to the simulation of deflection and load distributions on spur and helical gears covering both quasi-static and dynamic conditions. Li [10] investigated the effect of addendum on tooth contact stress and bending stress of spur gears. Tooth load, load sharing rate, transmission error and mesh stiffness were also analyzed. Arafa and Megahed [11] presented a FE modeling technique to evaluate the mesh compliance of spur gears, and discussed the load sharing among the mating gear teeth. Pimsarn and Kazerounian [12] developed a new method (pseudo-interference stiffness estimation) for evaluating the equivalent mesh stiffness and the mesh load in gear system.

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Finally, Vedmar [13] published some calculations of load distribution on helical gear teeth, obtained from contact and bending deformations.

In previous works [14,15], the authors developed a new model of load distribution for external gear teeth, from the minimum elastic potential energy criterion. The elastic potential energy of a pair of teeth was calculated and expressed as a function of the contact point and the normal load. The load sharing among several spur tooth-pairs in simultaneous contact was obtained by solving the variational problem of minimize the total potential (equal to the addition of the potential of each pair at its respective contact point), regarding the restriction of the total load to be equal to the sum of the load at each pair. The same approach was used for helical gear teeth by dividing each helical pair in infinite slices, perpendicular to the gear axis, assuming each slice to be equivalent to a spur gear with differential face width, and extending the integrals to the complete line of contact. This general model was applied to conventional external gears, including undercut gears [14], and high transverse contact ratio gears [15]. From these specific models, the location of the points of critical stress and the determinant load conditions were calculated for conventional [16,17], undercut [18] and high transverse contact ratio [19] external, spur and helical gears.

Much less developed are the strength calculation models for internal gear transmissions, with external tooth pinion and internal tooth wheel. Some studies on the mesh stiffness of internal spur gear pairs have been recently published [20,21] but a systematic study on the load distribution and tooth load capacity cannot be found in literature. Gear Standards [1–4] present some calculation methods of pitting and bending load capacity, parallel to those for external tooth gears, but not so well described and very weakly justified. Many rating factors are not specifically obtained for internal teeth, and may be unsuitable for internal gear transmissions. Among them, the load factors [1,3] cannot be equal to those for external teeth as the load distribution is not equal for internal and external gear transmissions.

In this work, a similar model of load distribution for internal gear transmissions is developed from the same hypotheses of minimum elastic potential energy. This load distribution model is used to determine the critical load conditions and the value of the corresponding critical stresses (bending stress and contact stress). The calculation of these critical stresses is performed according to the calculation methods of the nominal stresses of ISO 6336 [2–4], which probably will be not accurate enough for final designs but will be suitable for preliminary calculations or standardization purposes.

2. Load sharing model

The models presented in [14,15] for external gears are based on the assumption that the load sharing among the couples of teeth in simultaneous contact provides a minimum elastic potential. The development of the model is described in depth in [14]. In this section the model for internal gears will be briefly presented, to provide the background of the development of strength models presented in the following sections.

2.1. Load distribution for internal spur gears

The elastic potential energy of a spur tooth U can be expressed as the sum of the bending component U_x, the compressive component U_n and the shear component U_s:

$$U = U_x + U_n + U_s. \tag{1}$$

All the components can be computed from the equations of the theory of elasticity and the geometry of the tooth by considering the spur tooth as a cantilever beam of rectangular variable section, embedded at the root section and loaded at the contact point [14]. From these assumptions, the equations of the theory of elasticity result in:

$$\begin{aligned}
U_x &= \pm 6\frac{F^2}{Eb}\int_{y_p}^{y_C}\left[\pm(y_C-y)\cos\alpha_C - r_c\sin\frac{\gamma_c}{2}\sin\alpha_C\right]^2\frac{dy}{e^3(y)}\\
U_n &= \pm\frac{1}{2}\frac{F^2}{Eb}\int_{y_p}^{y_C}\sin^2\alpha_C\frac{dy}{e(y)}\\
U_s &= \pm C_s\frac{1}{2}\frac{F^2}{Gb}\int_{y_p}^{y_C}\cos^2\alpha_C\frac{dy}{e(y)}
\end{aligned} \tag{2}$$

where F is the normal load between both teeth, α_C the load angle, r_c the radius of the contact point, γ_c the angular thickness of the tooth at the contact point, b the face width, E the modulus of elasticity of the material, G the transverse modulus of elasticity, and $e(y)$ the tooth thickness at the section described by y, being y the coordinate along the tooth centerline from the gear rotation center. y_p and y_C are the values of coordinate y corresponding to the embedded section (defined by the points of both sides of the profile at the dedendum circle) and to the load section (the contact point section), respectively. Finally, C_s is the shear potential correction factor, which accounts the non-uniform distribution of the shear stresses on the section, according to the Colignon's theorem. For rectangular section, this factor takes the value $C_s = 1.5$. Finally, the plus/minus signs correspond to the external tooth pinion and the internal tooth wheel, respectively. Fig. 1 represents these geometrical parameters of the involute external and internal teeth.