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Power flow and efficiency analysis of multi-flow planetary gear trains

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A B S T R A C T

Power flow and efficiency analysis play an important role in the design of multi-flow planetary gear trains. Based on hypergraph and matrix operation, this paper proposes a new method to calculate gear speed ratio, velocity, torque and power. Systematic efficiency computation is carried out by following the power flow, and power loss equations on each node are derived via an approach based on self-rotation relative power. Power flow analysis and efficiency formulas are verified by two examples of multi-flow planetary gear trains. The proposed method significantly reduces the time required to establish the governing equations. Efficiency on each node can be calculated by following power flow, therefore making system efficiency calculation more accurate. Since power losses on certain nodes are dominant, parameter analysis is performed to analyze their influences.

1. Introduction

Multi-flow planetary gear trains are applied widely in vehicles, air planes and automation industry [1–3]. It is critical to determine the velocity relations, internal torques and power flow of a multi-flow planetary gear train, because firstly, velocity analysis is a basis for gear ratio and life cycles calculation, secondly, excessive internal torque may reduce the strength and stiffness of some components and thirdly, internal power flow, which plays an important role in accurate efficiency evaluation, may yield significant power loss to fail the design. Efficiency in planetary gear trains was reported lower than conventional gear trains for the high tooth load and relatively high meshing speed of the teeth [4]. Power losses in gear trains mainly come from gear mesh, oil churning, bearing and windage [5,6]. In this paper, it is assumed that the only power loss in a multi-flow planetary gear train is from gear meshing and planet bearings.

Different graphic methods have been used in power flow analysis in the literature [3,7–10]. Esmail and Hassan [3] established a block based on a basic gear pair. The block is connected by arrows and joints which represent different elements relative to the gear pair. A two degree-of-freedom (DOF) hybrid vehicle transmission system was analyzed using this method. Apparently this method has to deal with all the gear pairs to give the relations between each element in power flow analysis. Chen [7] analyzed the power flow of a power-split planetary gear train using a virtual power flow graph. This method is more accurate in the efficiency calculation, but it has to deduce complex equations of virtual power for different gear trains. Nelson and Cipra [8] employed graph theory to analyze the velocities of bevel epicyclic gear trains and the method is more suitable to software implementation. del Castillo [9] pointed out that even though the kinematic analysis of mechanisms using graphs has been widely developed by many researchers, a clearer way of understanding the relation between the components and power flow in complex planetary gear trains is still in need. In line with this idea, Goma Ayats et al. [10] applied hypergraphs to analyze the velocity relations of one DOF gear trains consisting of

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planetary gear train and hybrid gear trains consisting of planetary gear train and fixed-axis gear trains [11]. This method was much clearer and easier to use than other methods in velocity relation analysis. But it was limited to one DOF system and its kinematic equation analysis.

Efficiency calculation of different kinds of gear trains has been researched in other published works [12–17] including simple and compound planetary gear trains [18–22]. The efficiency equations are based on torque and power balance of the whole system, and the independent overall efficiencies were taken into consideration by these researches. But the power actually flows through each gear train component sequentially from input to output, thus the efficiencies on all components are in fact coupled together. Besides, a small amount of researches have tried to calculate the efficiency according to power flow. In Ref. [23], power flow patterns of simple one-DOF planetary gear trains were detailed. Chen [24] significantly simplified procedures of power analysis by introducing concepts of virtual motors and generators into virtual power analysis. After that, Chen [7] introduced two new relations: the split-power ratio and the virtual split-power ratio for the power-split planetary gear train analysis. However, these methods require complex formulas and are limited to certain specific gear trains, and it's not easy to integrate them in software.

This paper focuses on power flow and efficiency analysis of multi-flow planetary gear trains. A new method is proposed to calculate velocities, internal torques and power flow based on hypergraph and matrix operation, and a more accurate efficiency calculation is carried out by following power flow. This method can spare complex formula derivation and it is particularly suitable for software implementation. The rest of the paper is organized as follows: hypergraphs of multi-flow planetary gear trains are detailed in Section 2; the power flow analysis method based on hypergraph and matrix operation is presented in Section 3; formulas for efficiency calculation by following power flow are investigated in Section 4; two examples are analyzed in Section 5 by the proposed method; conclusions are summarized in Section 6.

2. Multi-flow planetary gear trains and their hypergraphs

The gear trains shown in Fig. 1 are two types of multi-flow planetary gear trains (MFPGT) which characterize: multiple DOF, power split and power confluence. Fig. 1(a) is a MFPGT applied in a hoist [2], which has two-DOF, two inputs and one output. Fig. 1(b) is a MFPGT applied in a vehicle transfer gearbox [1], which has two-DOF, one input and two outputs. Both examples are complex planetary gear trains and the power passes through multiple simple planetary gear trains (SPGT), shown in Fig. 1(a) as SPGTs *a*, *b* and *c*.

A simple planetary gear train as shown in Fig. 2(a) typically includes three shafts (one carrier, two central gears) and three planets. A basic triangle hypergraph (BTH) is used to represent the SPGT as shown in Fig. 2(b). In the BTH, the round node represents a shaft in the mechanism, and the line represents the mechanical connection between two shafts. The strategies for the symbols of a BTH and its nodes are given as follows: a BTH is named by a letter, such as *a* in Fig. 2(b). The numerals 1 (sun gear) and 3 (ring gear or sun gear) following a BTH's single-letter name represent the gear shafts, thus the combination *a*1 represents gear 1 in SPGT *a*; whereas numeral 2 following a BTH's name represents the carrier in a BTH, such as *b*2 represents carrier 2 in SPGT *b*. The letter *p* following a BTH's name represents the planet gear, and *ap* represents the planet in SPGT *a*. In this logic, a node can represented by two symbols such as *ij* or *mn* as a general symbol, where the first symbol, such as *i* or *m*, represents the BTH's name (*a*, *b*, *c*, …) and the second symbol, such as *j* or *n*, represents the number symbol (1, 2, 3) of the node, and the subscript *ij* or *mn* has the same meaning. The gearing ratio is put on an arrow and the direction of the arrow is from the active gear to the passive gear. The letter *k* with a BTH's name as a subscript is the ratio between gearing shaft 1 and 3 when carrier shaft 2 is fixed, that is

$$k_a = sign(\rho_a)$$

where *a* is the BTH's name, ρ is the characteristic parameter of the SPGT, $\rho_a = Z_{a3}/Z_{a1}$, and sign() indicates the sign of *k*, which is determined by the mesh types between planets and central gears.

In order to analyze the ratio, velocity, torque and power flow of the systems in Fig. 1, multi-flow planetary gear trains are separated into several simple planetary gear trains, and their BTHs are connected together to form hypergraphs of the systems shown in Fig. 3. In Fig. 3, the arrow directing into a node represents system input and the arrow directing out of a node represents system output. In the MFPGTs, it is typical that shafts in different SPGTs are connected together. Although the connected shaft has the same velocity in all relative SPGTs, the torque in each SPGT is different. Therefore the connected shafts are denoted by multiple overlapped circles which

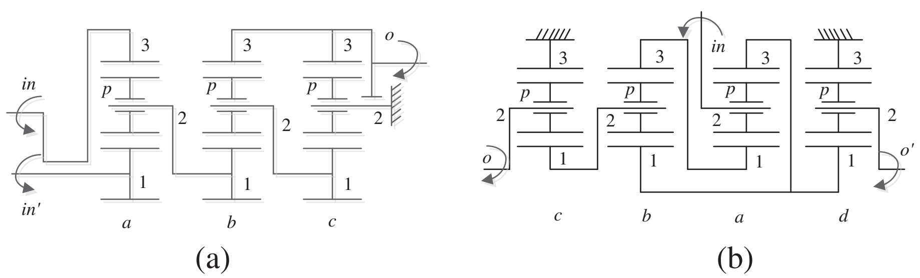

Fig. 1. Multi-flow planetary gear trains: (a) two-DOF, two inputs and one output and (b) two-DOF, one input and two outputs, where *in*, *in*′ represent inputs and *o*, *o*′ are outputs.