Mechanism and Machine Theory 92 (2015) 200–212

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

A method to determine kinematic accuracy reliability of gear mechanisms with truncated random variables

Xianzhen Huang *, Sen Hu, Yimin Zhang, Yichuan Xu

School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110819, China

ARTICLE INFO

Article history:
Received 20 May 2014
Received in revised form 31 December 2014
Accepted 28 April 2015
Available online xxxx

Keywords:
Gear mechanism
Kinematic accuracy
Mechanical reliability
Saddlepoint approximation

ABSTRACT

Gears are widely used as transmission components in various types of precision machinery, such as machine tools, vehicles and robots. The performance of these mechanical systems is directly related to the kinematic accuracy of the gear mechanism. This paper presents a novel method to evaluate the reliability of the kinematic accuracy of gear mechanisms with truncated random variables. The influence of errors originating from factors such as the gear eccentricity, profile deviation and assembly error on the transmission error are systematically analyzed and discussed. The statistical moments of the transmission error were obtained combined with the international standards for gear accuracy, tolerances, deviations and fits. A fit of the probability density function and cumulative distribution function of the transmission error was obtained using the saddlepoint approximation technique. A model was established to enable reliability analysis for gear mechanisms. Finally, a discussion of the practical applications along with a detailed analysis to evaluate the efficiency of the proposed method is presented.

1. Introduction

Gears are widely used as transmission components in various types of precision machinery, such as machine tools, vehicles and robots. The practical requirements for gear mechanisms may vary depending on the end goals of the user. However, the most basic requirement for these gear mechanisms used in mechanical systems is to transmit movement accurately and smoothly. If gears were perfect involutes, absolutely rigid and correctly spaced, the gear mechanism would work accurately and smoothly. However, this is not the case in practice due to the existence of various random factors such as gear eccentricity, profile deviation and assembly error. Random factors and variations in manufacturing are inevitable and errors in manufacturing cannot be completely eliminated. As a result, design engineers need to simulate the influence of random factors while taking functional requirements into consideration to evaluate the impact of the transmission performance for the gear mechanism.

Transmission error seriously affects the stability and accuracy of transmission and is considered to be an important characteristic to determine transmission performance of the gear mechanism. Recently, with the increasing requirement for precision manufacturing, transmission performance of gears has become more critical. Due to this growing trend, the effect of machining errors, assembly errors and tooth modifications on the transmission error of gear mechanisms is being investigated and verified either theoretically or empirically by various research groups [1–3]. Transmission errors can be minimized by using optimization procedures to define optimal profile modifications in spur and helical gears [4,5]. Reference [6] presented another optimization procedure, which is aimed at defining robust profile modifications with regard to transmission error variations in spur and helical gears. However, a vast majority of these studies were essentially deterministic and did not account for the transmission deviations caused by random factors in the manufacturing process.

* Corresponding author. Tel.: +86 24 83671429; fax: +86 24 83679731.
E-mail address: xzhhuang@mail.neu.edu.cn (X. Huang).

http://dx.doi.org/10.1016/j.mechmachtheory.2015.04.017

Kinematic accuracy analysis is routinely used with great success in manufacturing and assembly environments to verify critical dimensions. Predicting a stack-up of the contributing component variations controls these dimensions. Noting that geometric and assembly variations can cause variations in functionality; various researcher groups have focused on kinematic accuracy analysis [7–10]. Here, kinematic reliability is defined as the probability that a mechanism realizes its required motion path or trajectory within a specified tolerance range. Kinematic reliability accuracy of planar linkages or robot manipulators using the Monte Carlo simulation method [11], first order second moment method [12], hybrid dimension reduction method [13], and the CrossEnt method [14,15] have been reported in literature. However, to the best of our knowledge there are no reports in literature, which have examined the reliability of the kinematic accuracy specifically for the gear mechanism.

In this paper, a practical analytical method to determine the kinematic accuracy reliability of gear mechanisms is proposed. This is achieved by integrating theories of mechanical reliability and gear accuracy. We start with a brief description of the influences of random factors, such as gear eccentricity, profile deviation and assembly error, on the transmission accuracy of the gear mechanism. This is followed by calculation of the statistical moment of the transmission error. Next, an analysis of the kinematic accuracy reliability of the gear mechanism is presented. Finally, accuracy and efficiency of the proposed method were analyzed.

2. The transmission error of the gear mechanism

Transmission error of a gear mechanism can be described as the difference between the actual rotation angle and the theoretical rotation angle of the driven gear while the driving gear rotates. Transmission error for the gear mechanism is mainly caused by random uncertain factors in its components, such as manufacturing error for the gear, eccentricity of the shaft and assembly error.

For an involute cylindrical gear pair not affected by random uncertain factors, such as manufacturing and assembly errors, the relationship between the rotation angles of the driving shaft to the driven shaft should be linear. However, in practice, it is impossible to manufacture and assemble components of the gear pair without any errors. As a result, transmission errors cannot be completely eliminated in gear transmission.

2.1. Gear manufacturing error

Generally, gear manufacturing errors include gear eccentricity, tooth profile error, circular pitch error, tooth alignment error. Gear eccentricity (geometric eccentricity and motion eccentricity) takes one gear revolution as a period of change, which is called as the long periodic error. The gear eccentricity has direct influence on the kinematic accuracy of the gear mechanism. The tooth profile error, circular pitch error and tooth alignment error of the gear take one tooth rotation as a period of change and are referred to as the short periodic errors. The short periodic errors primarily affect the stability of the gear transmission.

2.1.1. Gear eccentricity

As depicted in Fig. 1, the driving gear has an integrated eccentricity $O_1O_1' = e_m$ (e_m is the vector sum of the geometric eccentricity and motion eccentricity) with respect to the rotation center O_1. φ_m is the angle between O_1O_2 and e_m. Due to the existence of the eccentricity (e_m), the meshing point of the gear pair is not B_1 but C_1. B_1C_1 is a segment of extra arc-length along which gear 1 rotates. The eccentricity e_m is much shorter than the pitch radius of the gear (r_1), therefore the extra arc-length along which the driven gear rotates is $B_1C_1 = B_2C_2 \approx e_m \sin \varphi_m$. Therefore, the gear pair rotates forward an extra arc-length, $e_m \sin\varphi_m$, (measure on the pitch circle) due to the influence of the gear eccentricity.

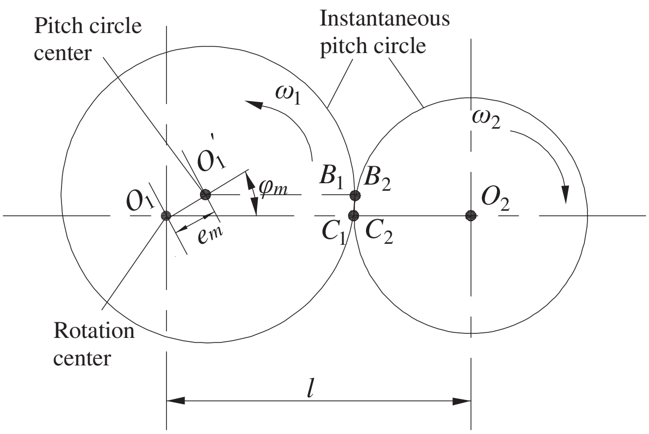

Fig. 1. Schematic illustration of the gear mechanism showing the parameters that influence gear eccentricity in a gear transmission.