Mechanism and Machine Theory 92 (2015) 407–435

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmt

Dynamic modeling and robust nonlinear control of a six-DOF active micro-vibration isolation manipulator with parameter uncertainties

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A R T I C L E I N F O

Article history:
Received 1 August 2014
Received in revised form 8 June 2015
Accepted 12 June 2015
Available online xxxx

Keywords:
Active micro-vibration isolation
Kane's method
Robust nonlinear
Parameter uncertainties
Stewart platform

A B S T R A C T

Dynamic model and vibration control of the Stewart platform are sophisticate and significant for vibration isolation especially the case with the base excitation. However, in the literature, no complete and accurate solution is presented. In this paper, a novel nonlinear dynamic model of six-spherical-prismatic-spherical (SPS) Stewart platform with the base excitation has been derived via Kane's method. Peculiarly, the dynamic model can be utilized for control algorithm designation to attenuate the base excitation. The order of the nonlinear coefficients has been evaluated to simplify the model. The uncertainties of stiffness, damping and mass center location have been studied based on the practical situation. An improved robust nonlinear controller, which is composed of the linear control part, nonlinear part and excitation compliment part, has been proposed to satisfy the two requirements at low frequency. The uniformly ultimate boundary of the state vector has been verified by theoretical proof accompanied with numerical simulation. It is concluded that the controller is of fine adjustability and can isolate a wide range of external disturbance.

1. Introduction

Optical satellites must deal with various disturbances on orbit [1]. For sensitive payloads, not being able to attenuate the disturbances to a sufficiently low level can either degrade the performance or cause malfunction. In order to satisfy the increasingly stringent requirements of optical payload on satellite, many manipulators that can be used for six Degree-of-Freedom (DOF) active vibration control have been conducted based on Stewart platform [1–7], which is of high load carrying capacity, good dynamic performance and precise positioning [8,9]. However, dynamic models of the platform are not accurate enough to meet the high standards of vibration isolation at low frequency. For instance, parameter uncertainties have not been conceived let alone the nonlinear properties of the platform.

The methods of dynamic modeling can be primarily divided into two categories: vector method and energy method, both of which lead to scalar equations but have different starting points [10]. Vector method starts with vector equations, including Newton–Euler method, momentum principles, D'Alembert principle and Kane's method. Energy method begins with scalar energy expressions which use Lagrange's equations, Hamilton's canonical equations, the Gibbs equations, or the Boltzmann–Hamel equations [11]. Dynamic equations formulated via Newton–Euler method [12–15] would suffer a heavy computational burden since some constraint forces are needed to be known. But the method is relatively easier to be understood and more direct than others. The Lagrange's equation [16–20] can be employed to analyze the dynamic characteristics of the platform. The fundamental issue is the selection of

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http://dx.doi.org/10.1016/j.mechmachtheory.2015.06.008

general coordinates and the expression of the kinematics. Once these are done, the rest is fairly straightforward. Although the forces of the joints are avoided, partial differentials of the kinematics with respect to general coordinates would cause additional computational complexity of the equations. Compared with these two aforementioned methods, Kane's method [10,11,21–25] which appears to be distinctly advantageous for multi-body problems, leads to concise and intuitive dynamic equations [11]. Another characteristic of Kane's method is that it's quite straightforward for the forward dynamics problems [29]. There are additional methods such as generalized momentum [26–28], virtual work [29] and screw theory [30].

Dynamic model of the Stewart platform is sophisticate and important for vibration isolation especially the case with the base excitation. However, in the literature, no complete and concise solution is presented. If the base excitation exists, the lower platform can no longer be regarded as an inertial reference. Therefore, we have to set an independent coordinate and calculate the transformation matrices from the local frames to the inertial one, which will bring additional complexity into the dynamic model. Besides, since the payload will be mounted on the upper platform, the disturbances can be transmitted to the payload directly without isolation. Consequently, the dynamic model with the base excitation is essential on which proper control algorithm design will be based. In this paper, we systematically present the dynamic equations with the base excitation through Kane's method. After a profound investigation of the actuator embedded in the link of the platform, it can be modeled as a piecewise linear spring which will bring structural nonlinearity into the equations. The new dynamic equation is concise and complete. Moreover, it has explicitly physical meaning.

Apart from the modeling methods, a suitable control strategy is also significant to multi-DOF vibration control. Generally, the control strategies [31] can be divided into passive control [32,33], active control and semi-active control [34]. Passive vibration isolation, which is effective for high frequency, is simple, reliable and stable. But it is not suitable for attenuating low frequency vibration because two trades-offs potentially exist in the passive vibration system [35]. Active vibration control can address these two limitations. To meet the actual situation, parameter uncertainties must be taken into account. As for the linearized model, classical robust control method [36–41] such as H∞ and μ-synthesis can be used to handle these problems [39]. However, neither H∞ control nor μ-synthesis can be applied to nonlinear dynamic systems. Robust nonlinear controllers based on Lyapunov function can be utilized as an alternative, which has been designed for nonlinear tracking control and active vibration control in both task space and joint space [42–49]. These controllers are based on the lemma given in [42] and [43]. Even though some work has been conducted on this topic, they are far from perfect. Taking an example of the tracking controllers proposed by Kim et al. [45,46], there is a room for improvement when it is utilized for vibration attenuation, which will be illustrated in this paper. Another one is presented by Yang et al. [47,48]. The main idea of the controller is to decrease the stiffness matrix as an identity matrix and increase the value of the damping matrix as much as possible, which seems to be hard to realize in practice.

For the sake of satisfying the two basic requirements in the ultra-low frequency domain, we have developed an improved robust nonlinear controller by introducing a linear controller and two weighting matrices for the purpose of vibration isolation. The proposed controller is composed of linear control part, nonlinear part and excitation compliment part. The linear one can be determined by linear control algorithms, such as PID, LQR, pole placement, etc. The nonlinear one compensates the nonlinear and the uncertain components. As for the excitation part, it compensates for the effect of base excitation. That the closed loop system can achieve uniformly ultimate boundary has been proved in this article. In addition, the performance conditions of the system can be achieved. Likewise, numerical simulations in the time domain demonstrate the fine performance of the controller. We assume that all the six links are identical throughout the paper.

2. Dynamic model with base excitation

In this section, the dynamic model of the platform with the base excitation will be formulated step by step. The platform has six DOFs including three displacements and three rotations. To start with, the kinematics of one link is analyzed. Then the partial velocities and the angular velocities are derived in the matrix form. Thirdly, dynamic equations with base excitation are derived. Fourthly, the axial force generated by the actuator is modeled and analyzed. Finally, the derived dynamic model is simplified into a concise form with the geometrical nonlinearities omitted.

2.1. Kinematics analysis of one link

In kinematics analysis, all the vectors are expressed in the inertial coordinate. Fig. 1 depicts the inertial frame **O**, local frame **B** and **P** that are fixed in the center of the lower and upper platform. The origins of local frames fixed in the sliding and cylindrical links are also shown in the figure. Vectors of the six links are denoted as $\mathbf{l}_i$ $(i = 1,\cdots,6)$

$$\mathbf{l}_1 = \overrightarrow{b_{16}p_{12}}, \mathbf{l}_2 = \overrightarrow{b_{23}p_{12}}, \mathbf{l}_3 = \overrightarrow{b_{23}p_{34}}, \mathbf{l}_4 = \overrightarrow{b_{45}p_{34}}, \mathbf{l}_5 = \overrightarrow{b_{45}p_{56}}, \mathbf{l}_6 = \overrightarrow{b_{16}p_{56}} . \quad (1)$$

The transformation matrix from frame **B** to inertial frame **O**,

$$\mathbf{R}_{(B/O)} = Rot(x,\alpha)Rot(y,\beta)Rot(z,\gamma) = \begin{bmatrix} c\beta c\gamma & -c\beta s\gamma & s\beta \\ c\alpha s\gamma + s\alpha s\beta c\gamma & c\alpha c\gamma - s\alpha s\beta s\gamma & -s\alpha c\beta \\ s\alpha s\gamma - c\alpha s\beta c\gamma & s\alpha c\gamma + c\alpha s\beta s\gamma & c\alpha c\beta \end{bmatrix} \quad (2)$$

where c and s denote cos and sin, respectively. α, β and γ are the *Cardano* angles of the lower platform.