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Mesh stiffness calculation using an accumulated integral potential energy method and dynamic analysis of helical gears

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ABSTRACT

As one of the most important dynamic excitation sources, the gear time-varying mesh stiffness is the key parameter of gear system dynamic model. So how to calculate the gear mesh stiffness accurately is of great importance. In this paper, an accumulated integral potential energy method is proposed to calculate the mesh stiffness of helical gears. Compared with the finite element model (FEM) and the ISO standard, the proposed analytical method is verified as its results agree well with the others'. Meanwhile the effects of different helical gears parameters (e.g., helix angle, normal module) on mesh stiffness are studied. The results show that the fluctuation degree of mesh stiffness becomes smaller as helix angle or face width increases, by contrast the influence of normal module on the mesh stiffness shows an opposite trend. Furthermore, the mesh stiffness reduction of helical gears due to the tooth crack is quantified. Finally, a dynamic model of helical gear transmission is established to study the vibration characteristics of helical gears. The results show that the mesh stiffness plays a vital role in controlling the vibration of helical gears and has a significant effect on the vibration characteristic of helical gears with tooth crack.

1. Introduction

Due to the advantages of steady transmission, lower impact and noise, helical gear mechanisms are widely used in many kinds of rotating machineries to transmit motion and power. Many researchers have investigated different gear dynamic models to ascertain the dynamic behavior of gear-rotor systems [1]. Some of them [2–8] study the dynamic behavior of gear-rotor systems under the different operation conditions and parameters (e.g., friction, transmission error, eccentricity error, backlash, shaft bow and so on). Some [9–15] investigate the vibration characteristic of the gear system with gear faults or bearing faults. In addition, there are some others [16–20] who specially investigate the dynamic behavior of the helical gears or bevel gears.

The work mentioned above has shown that variations in gear mesh stiffness are the primary sources of vibration excitation in gear drives. The research of mesh stiffness has attached massive attention: A so-called potential energy method which was presented by Yang and Lin [21] can calculate the mesh stiffness conveniently and effectively. Liu et al. [22] used the potential energy method to analytically evaluate the mesh stiffness of a planetary gear set. Wu et al. [23] developed this method by introducing straight line crack into the model. Wan et al. [24] improved this method by considering the flexibility between base circle and dedendum circle. Based on potential energy method, Pandya and Parey [25,26] studied the effect of crack path on mesh stiffness under different gear parameters like contact ratio, pressure angle, fillet radius and backup ratio. Chaari et al. [27,28] derived an analytical formulation to calculate the mesh stiffness and studied the effect of spalling or tooth breakage on mesh stiffness. Howard et al. [6] used finite element model (FEM) to analyze the influence of crack on mesh stiffness. Hedlund and Lehtovaara [29] used FEM to evaluate gear mesh stiffness variation of

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helical gears pair. Gu and Velex et al. [30] presented an approximate formulae of the mesh stiffness for ideal solid spur and helical gears.

The previous work on gear mesh stiffness mainly focuses on spur gears. Studies about mesh stiffness of helical gears are not enough and thorough, especially for the study about the effects of gear fault on the helical gears mesh stiffness. In addition, researching the effects of varied mesh stiffness on vibration characteristics has an important significance for the vibration control of helical gears transmission.

The contents of this paper are organized as follows: Firstly, an analytical method named accumulated integral potential energy method is proposed to calculate the mesh stiffness of normal and faulty helical gears. The proposed analytical method is compared with ISO standard and FEM to validate its accuracy. The gear mesh stiffness with different helical gears parameters is studied. Then authors of this paper used the proposed method to quantify the reduction of mesh stiffness due to crack fault. Thirdly, a dynamic model of helical gears transmission is established to study the vibration characteristics of helical gears.

2. Mesh stiffness calculation

2.1. Mesh stiffness calculation of spur gears by using potential energy method

In gear pair systems, the coupling between the torsional vibration and lateral vibration is mainly caused by mesh stiffness, thus it is very important to calculate the mesh stiffness conveniently and effectively.

In the potential energy method [23], the total potential energy stored in the mesh gear system was assumed to include four components: Hertzian energy U_h, bending energy U_b, shear energy U_s and axial compressive energy U_a which can be used to calculate Hertzian mesh stiffness k_h, bending mesh stiffness k_b, shear mesh stiffness k_s and axial compressive stiffness k_a, respectively. According to material mechanics and elastic mechanical, U_h, U_b, U_s and U_a can be expressed as follows:

$$U_h = \frac{F^2}{2k_h} \quad \text{where} \quad K_h = \frac{\pi E L}{4(1-v^2)} \tag{1}$$

$$U_b = \frac{F^2}{2k_b} = \int_0^d \frac{[F_b(d-x)-F_a h]^2}{2EI_x} \mathrm{d}x \tag{2}$$

$$U_s = \frac{F^2}{2k_s} = \int_0^d \frac{1.2{F_b}^2}{2GA_x} \mathrm{d}x \tag{3}$$

$$U_a = \frac{F^2}{2k_a} = \int_0^d \frac{{F_a}^2}{2EA_x} \mathrm{d}x \tag{4}$$

$$I_x = \frac{1}{12}(2h_x)^3 L \tag{5}$$

$$A_x = (2h_x)L \tag{6}$$

where F represents the acting force by the mating tooth in the contact point. F_a, F_b are radial and tangential forces, G, E, L, v represent shear modulus, Young's modulus, the width of tooth and Poisson's ratio, respectively. I_x, A_x are the area moment of inertia and area of the section where the distance from the dedendum circle is x, the other parameters are shown in Fig. 1(a).

When two gears are meshing, the total energy can be obtained by

$$U = \frac{F^2}{2k} = U_h + U_{b1} + U_{s1} + U_{a1} + U_{b2} + U_{s2} + U_{a2} = \frac{F^2}{2}\left(\frac{1}{k_h} + \frac{1}{k_{b1}} + \frac{1}{k_{s1}} + \frac{1}{k_{a1}} + \frac{1}{k_{b2}} + \frac{1}{k_{s2}} + \frac{1}{k_{a2}}\right) \tag{7}$$

where k represents the total effective mesh stiffness, subscripts “1” and “2” indicate the driving and driven gear, respectively.

Besides the tooth deformation, the fillet-foundation deflection also influences the stiffness of gear tooth [27]. An effective method to calculate gear foundation elasticity was proposed by Sainsot, Velex and Duverger [31]. The fillet-foundation deflection can be calculated by:

$$\delta_f = \frac{F}{k_f} = \frac{F\cos^2\alpha_m}{EL}\left\{L^*\left(\frac{\mu_f}{S_f}\right)^2 + M^*\left(\frac{\mu_f}{S_f}\right) + P^*\left(1 + Q^*\tan^2\alpha_m\right)\right\}. \tag{8}$$