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Load distribution analysis of clearance-fit spline joints using finite elements

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ABSTRACT

Splines are widely used in mechanical drive systems to transfer rotary motion from an input to an output. Despite their wide application in rotating machinery, very little is known about their contact behavior and load distribution characteristics. In this study, a combined finite element and surface integral contact analysis model is employed in order to investigate load distribution along the spline interfaces. Three loading cases are considered: (i) purely torsional loading representing power transfer through two concentric shafts, (ii) combined torsional and radial loading representing a spur gear–shaft interface, and (iii) combined torsional, radial, and moment loading representing a helical gear–shaft interface. The effect of spline misalignment is investigated along with intentional lead crowning of the contacting surfaces. In addition to spur spline teeth, helical spline teeth are investigated. Influence of intentional mismatch of splines through a slight helix angle applied to the external spline is also investigated within a range of torque transmitted. Finally, the effects of manufacturing tooth indexing error on spline load distributions are quantified using the proposed model.

1. Introduction

Splines are widely used in mechanical drive systems to transfer rotary motion and torsion from a shaft to a gear or from a gear to a shaft. The main advantage of splined shafts over keyed shafts is their higher load carrying capacity that often represents better durability performance. Moreover, spline couplings allow for a certain amount of angular misalignment and relative sliding between their internal and external components. They can transfer axial, rotational and torsional load effectively in case of helical gear loading.

The most common failure modes observed in spline joints include surface wear, fretting corrosion fatigue, and tooth breakage. Ku and Valtierra [1] studied wear of misaligned splines experimentally, demonstrating a significant effect of misalignments on the wear of a spline. Brown [2] reported accelerated wear of involute spline couplings in aircraft accessory drives primarily due to spline misalignment and undesirable lubrication conditions.

These experimental studies on splines were instrumental in defining and documenting failure modes in spline interfaces. Yet their contributions to the understanding of spline failure mechanisms were limited without knowing the load distributions along the spline contact interfaces. Review of the literature reveals only a few analytical models on splines, all of which were limited to simple loading conditions due to complexity of the contact in spline interfaces. Volfson [3] proposed a rough estimation of contact force distribution along the axial direction of splines under pure torsion or pure bending loading conditions. Tatur and Vygonnyi [4] developed an analytical model to estimate torque distribution along the face width direction of spline teeth for the case when

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the spline joint carries pure torsion. They proposed that the running torque $m(z)$ along the axial direction of the spline be determined by the following differential equation:

$$m(z) = \frac{dT^e(z)}{dz} = c_\varphi\left[\varphi^i(z) - \varphi^e(z)\right] \tag{1}$$

where $T^e(z)$ is the shaft torque, c_φ is the torsional stiffness of the spline joint which is assumed to be a constant along the axial direction, and $\varphi^i(z)$ and $\varphi^e(z)$ are twisting angles of the internal spline and external spline respectively. This simplified model requires a user-defined spline torsional stiffness. Barrot et al. [5–7] formulated the spline tooth torsional stiffness in the model of Tatur and Vygonnyi [4] by analyzing spline tooth deflections due to bending, shear, compression and base rotation. They calculated the load distribution along axial direction of splines under pure torsion loading conditions. These analytical models provide an estimate of load distribution along the face width direction of the splines under simple loading conditions, but they fail to predict the load distribution across the spline tooth profile direction. Furthermore, they fall short of handling load distribution of splines under combined loading conditions as is the case for gear–shaft spline joints. Other complicating effects such as spline tooth surface modifications and spline tooth manufacturing errors such as indexing or spacing errors are also not considered in these models.

Another group of more recent studies proposed computational models of splines using the finite element (FE) method or boundary element (BE) method. FE models by Limmer et al. [8], Kahn-Jetter and Wright [9] and Tjernberg [10,11] used commercial FE packages to predict spline load distributions under pure torsion loading, while the last two accounted for effects of certain manufacturing errors as well. FE models for helical spline couplings were proposed by Leen et al. [12–14] and Ding et al. [15–17] for splines under combined torsional and axial loading. Adey et al. [18] developed a model using boundary element method for spline analysis. This model had the capability to analyze combined torsional and bending loading in the presence of certain manufacturing errors. Using Adey's model, Medina and Olver [19,20] studied load distribution of misaligned splines, and the impact of spline pitch errors and lead crown modifications.

The above literature review indicates that there is no widely accepted general analysis tool for spline load distribution. Most of these did not include combined radial, torsional and bending loading conditions experienced by spur and helical gear splines. Under such complex loading conditions, effects of spline tooth modifications and manufacturing errors are not fully understood. Consequences of the common practice of applying an intentional mismatch of splines also remain unknown. Furthermore, extensive parameter studies of the effects of spline tooth modifications and manufacturing errors are also not available. Accordingly, this paper aims at developing FE based computation model of gear–shaft splines. The objectives of this paper are as follows. (i) Develop a computational model of a gear–shaft spline interface under combined torsion, radial forces and tilting moments. (ii) Establish nominal load distribution conditions under pure torsion, spur gear loading (torsion and radial force) and helical gear (torsion, radial force and tilting moment) loading conditions. (iii) Quantify the change to baseline load distributions caused by misalignments, spline tooth (lead and profile) modifications, spline helix angle and intentional helical mismatch. (iv) Investigate the influence of manufacturing errors on baseline spline load distributions.

2. Computational model

A commercial FE based contact mechanics model Helical 3D (Advanced Numerical Solutions, Inc.) designed specifically for loaded contact analysis of helical gears is modified here to analyze spline joints. An efficient and accurate finite quasi-prism (FQP) element [21] is used in this model to represent spline surfaces. The core contact solver of this software (CALYX) is based on a formulation by Vijayakar [22], which combines the finite element method and surface integral method to represent the contact bodies, and calculates the load distribution and rigid body displacements by using the linear programming method.

The first phase of contact analysis is to determine the contact zone. CALYX estimates the contact zone by using Hertz's model after locating a set of "primary contact points" on the contacting surfaces and determining relative principal curvatures and directions. The second phase is to compute the compliance matrix and set up the contact equation to be solved by a modified simplex method.

The first step in the process is locating the primary contact point. For this, two contacting surfaces Σ_1 and Σ_2 are defined in terms of their curvilinear parameters s and t as $\mathbf{r}_1(s_1,t_1)$ and $\mathbf{r}_2(s_2,t_2)$. The primary contact points are determined and located when $\mathbf{r}_1$ and $\mathbf{r}_2$ become the closest to each other [22]. For this, the surface $\mathbf{r}_1(s_1,t_1)$ is discretized into a grid of points $\mathbf{r}_{1ij}(s_{1i},t_{1j})$. For each of these grid points, a primary contact point $\mathbf{r}_{2ij}(s_{2i},t_{2j})$ is determined such that $\|\mathbf{r}_1(s_1,t_1) - \mathbf{r}_2(s_2,t_2)\|$ is minimum. This extremization is equivalent to solving the following system of nonlinear equations [22]:

$$\begin{cases} \left[\mathbf{r}_{1ij} - \mathbf{r}_2\left(s_{2i}, t_{2j}\right)\right] \cdot \dfrac{\partial \mathbf{r}_2\left(s_{2i}, t_{2j}\right)}{\partial \mathbf{s}_{2i}} = 0 \\ \left[\mathbf{r}_{1ij} - \mathbf{r}_2\left(s_{2i}, t_{2j}\right)\right] \cdot \dfrac{\partial \mathbf{r}_2\left(s_{2i}, t_{2j}\right)}{\partial \mathbf{t}_{2j}} = 0 \end{cases} \tag{2}$$

Newton–Raphson method is used to solve this system of non-linear equations to obtain mating points $\mathbf{r}_{2ij}$ on the second surface for each of the grid points $\mathbf{r}_{1ij}$. Then a refined grid is set up around the point $\mathbf{r}_{1ij}$ such that the separation $\|\mathbf{r}_{1ij} - \mathbf{r}_{2ij}\|$ is the smallest. This process is repeated several times with progressively smaller grids to locate the principal contact point p [22].