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# Variational tolerancing analysis taking thermomechanical strains into account: Application to a high pressure turbine

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ABSTRACT

The aim of this study is to propose a variational method of tolerancing analysis using a multiphysical approach. This method is based on operations on polytopes (Minkowski sum and intersection) and can be used to validate geometric specifications, contact specifications and thermomechanical specifications.
The first part describes how thermomechanical strains are integrated into a tolerancing analysis tool, based on operations on polytopes. In the second part correlations are defined between two turbine performance criteria, leakage section and risk of touching, and two geometric conditions respectively.
In the third part, the influence of design choices is described, in particular the influence of the shape of the parts and the behaviour of the joints on the thermomechanical operating regime of the turbine.
Two turbine architectures are considered in relation to the same two performance criteria, and lastly the main turbine architecture results are discussed and future developments are described.


## 1. Introduction

Controlling the behaviour and the energy yield of turboshaft engines for each of the different operating regimes is essential to ensure that the desired power is achieved. One way to improve the performance of these turboshaft engines is to control the geometric variability of the turbine, and more particularly the clearance between the blade tips and the stator.

In the preliminary design phase, several alternative turbine architectures are envisaged. These alternatives are often based on different component shapes and dimensions, with several technical solutions being proposed for joints between components and different materials. In this article we propose a model which will define clearance between blade tips and stator for different turbine architectures, taking the following variabilities into account:

– processes for obtaining parts,
– processes for assembling parts,
– thermomechanical behaviour of the turbine.

The model described in this article is based on a variational approach, manipulating sets of geometric constraints that formalise polytopes [1].

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This type of approach does support the redundancy of suppressing degrees of freedom between two parts. The integration of the redundancy of suppressing degrees of freedom between two parts is trivial in finite element methods for thermomechanical simulations but not in tolerancing analysis methods [2]. For example, Ref. [3] presents a tolerance analysis method taking into account thermal dilatation, but this work does not integrate thermomechanical deviations of parts dependent on contact conditions with other parts. The main reason is that the tolerancing analysis tool used in Ref. [3] does take into account the redundancy of suppressing degrees of freedom.

A polytope is the bounded intersection of a finite set of closed half-spaces of $\mathbb{R}^n$ of which the boundaries are hyperplanes of $\mathbb{R}^{n-1}$ [4]. A polytope is used to define all the positions of a surface within a tolerance zone (geometric polytope) and all the relative positions between two surfaces potentially in contact (contact polytope). By applying operations (Minkowski sums [5,6] and intersections) to geometric and contact polytopes it is then possible to characterise the relative position between the rotor and the stator in a turbine. These operations are deduced from the topological structure of the turbine defined by a contact graph for one connected component [7].

There are models that characterise geometric variations by sets of constraints; these include the domain [8], the T-Map [9] and the polyhedron [10]. In contrast to the polytope, in domains and T-Maps the constraints are generally not linear.

The main motivation in using operations on polytopes is the complexity of Minkowski sums [4–6]. The complexity of Minkowski sums of sets of nonlinear constraints is prohibitive in $\mathbb{R}^6$.

The main originality of this method consists of integrating the thermomechanical deviations into the 3d dimension chains based on operations on polytopes.

Clearance between the rotor and the stator in the turbine is defined in terms of a reference model with a reference behaviour. In the reference behaviour, all the parts are at 20 °C and are modelled by infinitely rigid solids [11]. In the reference model any geometric variability due to manufacturing and assembly processes can be shown.

The turbine's operating cycle is discretized into several distinct thermomechanical behaviours which correspond respectively to different thermomechanical specifications [12]. No transitional regime is considered. Clearance between the rotor and the stator in the turbine is characterised by a thermomechanical model based on a reference model and incorporating thermomechanical strains in the parts and the contacts. The thermomechanical model is then used to define geometric variability due to manufacturing and assembly processes and also variability inherent in the thermomechanical behaviour of the turbine.

Two performance criteria are formulated with which to qualify the proposed turbine architectures: risk of touching and leakage section between rotor and stator. This work is part of a general series of studies into decision support systems to assist the designer in choosing a qualified turbine architecture that performs better than any other. In such context, simulations according to the worst case are relevant; it is not necessary to use probabilistic simulations.

In the first part, the procedure for modelling different geometric variabilities is described. In the second part the qualification criteria for turbine architectural solutions are presented. In the third part, we describe an application of this work to a sub-unit of a turbine of turboshaft engine.

Finally, after drawing the principal conclusions, future developments for this work are presented.

## 2. Modelling geometric variability

The geometric variability in processes for obtaining parts and in assembly processes are taken into account by 3d dimension chain simulation tools. However, most of these tools model the different parts as infinitely rigid solids, so to make up for this, the thermomechanical strains on the parts must be integrated into 3d dimension chain simulations. This is essential in order to control clearance at the tip of the turbine blade in different operating phases in a turboshaft engine [11].

### *2.1. Variability due to manufacturing and assembly processes*

The geometric models used in 3d dimension chains are generally based on the following hypotheses: no defect in the shape of the real surfaces, no local strain on surfaces in contact and no flexible parts. These hypotheses define the reference behaviour of a system. The limits for geometric defects in a part (defined by specification) are defined by a geometric polytope; the acceptable limits for relative displacement between two surfaces in contact (defined by clearance) are defined by a contact polytope. A functional condition expressed between any two surfaces of a mechanism is characterised by a functional polytope. Respecting a functional condition is simulated by including a calculated polytope in the functional polytope [1,13,14]. The calculated polytope is the result of operations (Minkowski sums and intersections) between geometric polytopes and contact polytopes.

Let us consider Fig. 1a: surfaces 1,2 and 1,3 of part 1 are in contact with surfaces 2,2 and 2,3 of part 2 respectively.

Contact between two surfaces, i.e. a joint, can be defined using a set of parameters. There have been several studies on this subject [15,16,2]. Hereafter we will use the definition proposed in Ref. [17], which is a direct application of that described in Ref. [2].

A joint is defined according to the following parameters:

- type: planar pair, cylindrical pair, ball and cylinder pair, etc.
- situation element(s): plane, straight line, point
- nature: fixed, sliding or floating
- clearance: minimal clearance and maximal clearance: $J_{min}$, $J_{max}$.