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Reconfiguration analysis of a 3-DOF parallel mechanism using Euler parameter quaternions and algebraic geometry method ☆

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A R T I C L E I N F O

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A B S T R A C T

This paper deals with the reconfiguration analysis of a 3-DOF (degrees-of-freedom) parallel mechanism (PM) with multiple operation modes – a disassembly-free reconfigurable PM – using the Euler parameter quaternions and algebraic geometry approach. At first, Euler parameter quaternions are classified into 15 cases based on the number of constant zero components and the kinematic interpretation of different cases of Euler parameter quaternions is presented. A set of constraint equations of a 3-RER PM with orthogonal platforms is derived with the orientation of the moving platform represented using a Euler parameter quaternion and then solved using the algebraic geometry method. It is found that this 3-RER PM has 15 3-DOF operation modes, including four translational modes, six planar modes, four zero-torsion-rate motion modes and one spherical mode. The transition configurations, which are singular configurations, among different operation modes are also presented. Especially, the transition configurations in which the PM can switch among eight operation modes are revealed for the first time.

1. Introduction

Parallel mechanisms (PMs) with multiple operation modes [1,2] (also called PMs that change their group of motion [3], PMs with bifurcation of motion [4–6], or disassembly-free reconfigurable PMs [2]) are a novel class of reconfigurable PMs which need fewer actuators and less time for changeover than the existing reconfigurable PMs. Several classes of PMs with multiple operation modes have been proposed in the past decade.

A PM with multiple operation modes was first applied in a constant-velocity coupling to connect intersecting axes in one mode or parallel axes in another mode [7]. Since DYMO – a PM with multiple operation modes – was proposed in [8], several new PMs with multiple operation modes have been proposed [1–6,9]. How to switch a PM with multiple operation modes from one configuration to another requires reconfiguration analysis in order to fully understand all the operation modes that the PM has and the transition configurations between different operation modes. This requires solving polynomial equations with sets of positive dimensional solutions. Recent advances in algebraic geometry [10,11] and numerical algebraic geometry [12] as well as computer algebra systems [13] provide effective tools to the reconfiguration analysis.

This paper aims to fully investigate the operation modes of a 3-RER PM and the transition configurations to switch from one operation mode to another. Here, R and E denote revolute and planar joints respectively. In Section 2, Euler parameter quaternions will be classified based on the number of constant zero components. The kinematic interpretation of different cases of Euler parameter quaternions will be discussed. In Section 3, the description of a 3-RER PM with orthogonal platforms will be presented. In Section 4, by representing the position and orientation of the moving platform using the Cartesian coordinates of a

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point on the moving platform and a Euler parameter quaternion respectively, a set of kinematic equations of the 3-RER PM will be derived and then solved using the algebraic geometry method to obtain all the operation modes of the PM. The transition configurations among different operation modes will be obtained in Section 5. Finally, conclusions will be drawn.

2. Classification of Euler parameter quaternions

Euler parameter quaternions, which are more computationally efficient than the transformation matrices, have been used in kinematics, computer visualization and animation, and aircraft navigation [14,15]. In this section, we will first recall the definition and operation of the Euler parameter quaternions and then discuss the classification and kinematic interpretation of the Euler parameter quaternions.

The Euler parameter quaternion is defined as (Fig. 1)

$$q = e_0 + e_1\mathbf{i} + e_2\mathbf{j} + e_3\mathbf{k} = cos(\theta/2) + \mathbf{u}\,sin(\theta/2) \tag{1}$$

where $\mathbf{u}$ and θ represent respectively the axis[1] and angle of rotation. The Euler parameters satisfy

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1. \tag{2}$$

Let $q = e_0 + e_1\mathbf{i} + e_2\mathbf{j} + e_3\mathbf{k}$, the conjugate of q is

$$q^* = e_0 - e_1\mathbf{i} - e_2\mathbf{j} - e_3\mathbf{k}. \tag{3}$$

The product of two Euler parameter quaternions satisfies the following rules:

$$\begin{array}{c} \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1 \\ \mathbf{ij} = \mathbf{k} = -\mathbf{ji} \\ \mathbf{jk} = \mathbf{i} = -\mathbf{kj} \\ \mathbf{ki} = \mathbf{j} = -\mathbf{ik}. \end{array} \tag{4}$$

A vector $\mathrm{r} = \{r_x r_y r_z\}^T$ can be written in quaternion form as $\mathbf{r} = r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k}$. Let $\mathbf{r}' = r_x'\mathbf{i} + r_y'\mathbf{j} + r_z'\mathbf{k}$ denotes the vector obtained from a vector $\mathbf{r} = r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k}$ by rotating it about the axis $\mathbf{u}$ by θ. We have

$$\mathbf{r}' = q\mathbf{r}q^*. \tag{5}$$

The compositional rotation composed of rotation q_1 followed by rotation q_2 can be represented using the following quaternion

$$q = q_2 q_1. \tag{6}$$

From Eq. (2), one learns that e_0, e_1, e_2 and e_3 cannot be equal to zero simultaneously. Euler parameter quaternions can then be classified into the following 15 cases according to the number of their constant 0 components:

$$\begin{array}{l} \{e_0, 0, 0, 0\}, \{0, e_1, 0, 0\}, \{0, 0, e_2, 0\}, \{0, 0, 0, e_3\}, \{e_0, e_1, 0, 0\}, \{e_0, 0, e_2, 0\}, \\ \{e_0, 0, 0, e_3\}, \{0, 0, e_2, e_3\}, \{0, e_1, 0, e_3\}, \{0, e_1, e_2, 0\}, \{0, e_1, e_2, e_3\}, \{e_0, 0, e_2, e_3\} \\ \{e_0, e_1, 0, e_3\}, \{e_0, e_1, e_2, 0\} \text{ and } \{e_0, e_1, e_2, e_3\}. \end{array}$$

Using Eq. (1), we can identify the kinematic meaning of nine cases of Euler parameter quaternions (Nos. 1–7, 11 and 15 in Table 1) directly. For example, the Euler parameter quaternion of case $\{0, e_1, 0, 0\}$ is $q = \mathbf{i}$ (No. 2 in Table 1), which represents a half-turn rotation about the X-axis. The DOF of the above motion is 0 and the angular velocity is 0. The Euler parameter quaternion of case $\{e_0, e_1, 0, 0\}$ is $q = e_0 + e_1\mathbf{i}$ (No. 2 in Table 1), which represents a rotation by $2atan(e_1, e_0)$ about the X-axis. The DOF of the above motion is 1 and the axis of the angular velocity is the X-axis. The Euler parameter quaternion of case $\{0, e_1, e_2, e_3\}$ is $q = e_1\mathbf{i} + e_2\mathbf{j} + e_3\mathbf{k}$ (No. 11 in Table 1). Kinematically, it refers to a half-turn rotation about the axis $\mathbf{u} = \{e_1\ e_2\ e_3\}^T$. Since the angle of rotation about the axis $\mathbf{u} = \{e_1\ e_2\ e_3\}^T$ is a constant, the component of the angular velocity along this axis is zero. The above motion can therefore be called a 2-DOF zero-torsion-rate rotation. It is noted that such a rotation is called a zero-torsion rotation in [16]. However, the torsion angle under the above motion is a constant and may not be zero depending on the mathematical representation of rotation.

For the remaining six cases of Euler parameter quaternions (Nos. 8–10 and 12–14 in Table 1), we cannot obtain explicitly the axis of angular velocity for a 1-DOF rotation and the axis along which the component of the angular velocity is zero for a 2-DOF rotation by using Eq. (1) directly. By factoring each of these six cases of Euler parameter quaternions as the product of two cases of Euler parameter quaternions (Nos. 1–7 and 11 in Table 1), one can identify their kinematic interpretation which reflects the motion characteristics using Eqs. (1) and (6).

[1] Throughout this paper, the axis $\mathbf{u}$ passes through the origin O of the reference coordinate system O-XYZ of a mechanism.