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# Dynamic analysis of a lubricated planar slider–crank mechanism considering friction and Hertz contact effects

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A B S T R A C T

This paper presents the development of a dynamic model for the slider–crank mechanism with clearance on the piston–pin revolute joint. The equations of motion for this system are obtained by Lagrange's method and the effects related to contact, friction and lubrication at the elements that operate in the clearance are the targets of study. The contact force model used in this work is based on Hertz formulation, considering the inclusion of the dissipative effect associated with the impact between the pin and the piston. The frictional force adopted is based on the Coulomb friction but adapted to the multibody dynamics approach. Such models are verified with the results found in recent literature. The research presents contribution in evaluating the effect introduced by hydrodynamic lubrication in the revolute joint clearance. Two models of hydrodynamic lubrication are investigated: the first model presents a direct solution of low computational cost, the second model results in a numerical solution that consider the effect of the acceleration of the lubricant fluid imposed on the movement of the mechanism. It was observed that the second lubrication model does not guarantee the support of the piston–pin system for hydrodynamic lubrication in the simulated interval of time. Therefore, it is necessary to develop a more realistic model of hydrodynamic and elastohydrodynamic lubrication that is capable of reproducing the behavior of the piston–pin contact.



## 1. Introduction

The focus of the investigation presented here is the contribution in addressing the hydrodynamic lubrication in mixed models, where there is a transition between the hydrodynamic regime and the contact with friction dissipative effect. This is a typical problem in mechanisms with revolute joints with clearance, being one of the most significant applications of this problem, the slider–crank mechanism, widely used in machinery for transformation of rotational kinetic energy into kinetic energy of translation, or vice versa, for example, internal combustion engines and compressors. This topic has been object of research in the last decade [1,2], however, the approaches used to model the contact/friction and lubrication effects are still an open issue.

In order to analyze the dynamic behavior of mechanisms with lubricated revolute joints, a hydrodynamic lubrication model should be applied. In this kind of joint, the clearance is filled with a lubricant that generates a pressure distribution and, consequently, hydrodynamic forces actuating in the components of the mechanical system. Recently, some researchers have investigated the dynamic behavior of mechanisms considering the lubrication action at the clearance joints.

Schwab et al. [3] compared different revolute joint clearance models in the dynamic analysis of rigid and elastic mechanical systems. The hydrodynamic lubrication model, in this case, was based on the Reynolds equation for a thin film, incorporating the finite length of the bearing and the effect of cavitation in the fluid film.

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In 2004, Flores et al. [4,5] accomplished a dynamic analysis of a mechanical system with lubricated joints. Thus, a hydrodynamic lubrication model was applied, considering the infinitely long journal bearing condition and hydrodynamic forces due to only squeeze effects. However, for high angular velocities, the simple squeeze approach is not valid and, consequently, the Reynolds equation should be applied. For this reason, Flores et al. [6] continued the study on dynamics of mechanical systems including joints with clearance and lubrication. In this work, a hydrodynamic lubrication model based on the solution of the Reynolds equation was proposed, considering the infinitely long journal bearing condition. Moreover, the hydrodynamic forces were obtained due to the squeeze and wedge effects.

Flores [7] also presented a methodology to modeling wear in revolute clearance joints in multibody systems. In this study the nonuniformity of the wear depth along the joint surface was verified due to the fact that the contact between journal and bearing walls is wider and more frequent in some specific regions.

In 2009, Flores et al. [8] presented a general methodology for modeling lubricated revolute joints in constrained rigid planar multibody systems. In this work, hydrodynamic lubrication models were presented considering the infinitely long and short journal bearing conditions. Spatial rigid-multibody systems with lubricated spherical clearance joints were also studied by Flores and Lankarani [9] in 2010.

Still in 2009 Flores et al. [10] investigated a numerical and experimental response of the slider–crank mechanism with revolute clearance joint. The results obtained from the numerical model had a good agreement with the experiment, pointing some critical parameters like joint properties, friction and restitution coefficients that influence the outcome of the numerical results.

Muvengei et al. [11] presented interesting results considering the multibody systems with differently located frictionless revolute joints with clearance, and the LuGre friction force model [12] to capture and characterize slip–stick effects.

Tian et al. [13] simulated the behavior of planar flexible multibody systems with clearance and lubricated revolute joints. For lubricated revolute joints, the hydrodynamic forces were calculated as proposed by Flores et al. [8]. However, aiming to consider possible cavitation effects, some modifications were done in the lubrication model as proposed by Ravn et al. [14].

Daniel and Cavalca [15] analyzed the dynamic of a slider–crank mechanism with hydrodynamic lubrication in the connecting rod–slider joint clearance. The hydrodynamic lubrication model used for this analysis considered the infinitely long journal bearing condition and the hydrodynamic forces were obtained due to only wedge effect. This model was previously developed by the authors in Bannwart et al. [16] and it was based on the integration of the differential equations of mass flux and momentum, considering also the acceleration of the lubricant due to alternate translational motion of the bearing.

Machado et al. [17] studied the effect of the lubricated revolute joint parameters and hydrodynamic force models on the dynamic response of planar multibody systems. This way, three different hydrodynamic force models were considered, being the Pinkus and Sternlicht [18] model for infinitely long journal-bearings and the Frêne et al. [19] models for infinitely long and short journal-bearings. According the results obtained in the simulations, the hydrodynamic force models changed the dynamic characteristics of the multibody systems. Finally Tian et al. [20] presented and study developed for multibody approach considering the elastohydrodynamic lubrication in cylindrical joints. Hence, the lubrication model applied in the piston–pin system analysis should be carefully chosen.

Therefore, the authors developed a hydrodynamic lubrication model for bearings with alternate motion [16], which was applied in a planar slider–crank mechanism with a clearance in the three degrees of freedom conrod–slider joint [15]. The present study aims to investigate the performance of this model when combined with a contact model with dissipative effect (friction), well consolidated in the literature [21], obtaining, consequently, a better representation of the physical problem. To evaluate the results obtained from the hydrodynamic model proposed here [15,16] a comparison with a classic hydrodynamic lubrication model [18] was made, as well as the inclusion of the explosion external force effect in the dynamic model of the mechanism, observing its influence on the lubrication conditions with respect to the contact or hydrodynamic regime.

## 2. Dynamic model of the mechanism

### 2.1. Dynamic equations of motion

A planar slider–crank mechanism considered for analysis is shown Fig. 1. This mechanism is used in internal combustion engines and it is modeled to have a clearance at piston–pin revolute joint. This cylindrical joint attaches the connecting rod pin (colored in black) to the piston. While the pin can moves freely on a plane, the piston only moves in vertical direction. This way, the mechanism has three degrees of freedom (or independent co-ordinates): the crank angular displacement (q); the connecting rod angular displacement (A) and the slider vertical position ($X_{PT}$).

From Fig. 1 the following geometric parameters are defined: the crank length (R); and the length of the connecting rod (L). The position of the center of piston–pin ($X_P$,$Y_P$) and its velocity at inertial frame ($\dot{X}_P$,$\dot{Y}_P$) is calculated from expressions (1) and (2) respectively [22].

$$\begin{Bmatrix} X_p \\ Y_p \end{Bmatrix} = \begin{Bmatrix} R\cos(q) + L\cos(A) \\ R\sin(q) - L\sin(A) \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} \dot{X}_p \\ \dot{Y}_p \end{Bmatrix} = \begin{bmatrix} -R\sin(q) & -L\sin(A) \\ R\cos(q) & -L\cos(A) \end{bmatrix} \begin{Bmatrix} \dot{q} \\ \dot{A} \end{Bmatrix}. \tag{2}$$