ARTICLE IN PRESS

Precision Engineering xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Precision Engineering

journal homepage: www.elsevier.com/locate/precision

Prediction of chatter stability for milling process using precise integration method

Yuebang Dai, Hongkun Li*, Xinyu Xing, Baitian Hao

School of Mechanical Engineering, Dalian University of Technology, Dalian, 116024, China

ARTICLE INFO

Keywords:
Milling process stability lobe diagram precise integration method runge-kutta based complete discretization method complete discretization scheme with the euler method

ABSTRACT

The explicit precise integration method (PIM) is adopted to predict the chatter stability of milling process. To begin with, the dynamic equation with the regenerative effect is established from a simplified 2-DOF milling model. Then, the general solution for Hamilton system of the delay differential equation (DDE) is obtained by ordinary differential theory. To attain time series expression, the nonhomogeneous term is expanded by Taylor formula. Subsequently, the exponential matrix is resolved by 2^Nth order algorithm, which achieves complete discretization of iterative formula. Lastly, the stability lobe diagram is constructed by utilizing Floquet theory to judge the eigenvalues of the state transition matrix corresponding to certain cutting conditions. A classic benchmark example is introduced and the comparison results show PIM is valid for prediction of chatter stability.

1. Introduction

The cutting chatter caused by unsuitable cutting parameters in milling process has a seriously damaging effect on machining tools and workpiece precision. At present, the effective and important way to avoid chatter is to construct the stability lobe diagram (SLD) before machining, in which the relation of the critical axial cutting depth and the spindle speed under given cutting conditions can be expressed clearly.

Since Altintas and Budak [1] proposed the zero-order analytical (ZOA) method for constructing SLD based on the Fourier series of the dynamic modeling of milling process, the algorithms for the SLD have been developed continuously. Medrol et al. [2] expanded the ZOA method and proposed the multifrequency method (MF) for the low immersion milling problem by improving the transfer function with the harmonics of the tooth passing frequencies. The analytical methods above are fast and effective, but the various nonlinear factors in milling process are unable to be taken into modeling consideration, so that the additional stability regions cannot be predicted [8]. In order to overcome this problem, more and more scholars focused on the numerical solutions for differential dynamic equation (DDE) and considerable methods have been presented, among which the representative are as follows. Insperger et al. [3] presented the semi-discretization method (SDM), which only discretizes the delay term and keep lower computational efficiency for the calculation of the index matrix. Ding H et al. [4] interpolated the time delay term and time periodic part of time domain terms and presented a full-discretization method (FDM), and 2 nd FDM [5], which maintains higher prediction efficiency than SDM. Li M et al. [6] presented a complete discretization scheme with expanding all parts of DDE, including delay term, time domain term, parameter matrices by using Euler's method (CDSEM), which is efficient in comparison with FDM and SDM. Li Z et al. [7] firstly proposed three-order Runge-Kutta Method (RKM). Later, based on complete discretization scheme, they promoted a Runge-Kutta based complete discretization method (RKCDM) [8], which converges faster than CDSEM and SDM. The numerical differentiation method (NIM) is presented by Zhang X et al. [9], where the state terms are represented by a linear combination of multi-state function.

However, regardless of SDM, CDSEM, RKM, RKCDM, or NIM, the discretization error caused by difference scheme is difficult to be completely eliminated [10]. In this paper, on basis of classic precise integration method (PIM) [11], an explicit PIM obtained by expanding inhomogeneous term using Taylor formula is introduced to prediction of chatter stability for milling process, which ensures there is not any numerical accuracy loss. It is verified with a classic example. Hence, the rest of this paper is organized as follows. Section 2 shows the algorithm of PIM in detail. Section 3 makes the comparison of PIM and other numerical methods in convergence rate, computation efficiency and

* Corresponding author.
E-mail address: lihk@dlut.edu.cn (H. Li).

https://doi.org/10.1016/j.precisioneng.2017.12.003
Received 6 August 2017; Received in revised form 19 November 2017; Accepted 4 December 2017

Please cite this article as: Dai, Y., Precision Engineering (2017), https://doi.org/10.1016/j.precisioneng.2017.12.003

accuracy. Some conclusions are drawn in Section 4.

2. Precise integration method for prediction of chatter stability

2.1. Dynamics modeling of milling process

Just considering the vibration factors of cutter in the feed direction x and the normal direction y alone, the cutter-workpiece system is simplified into two degrees of freedom (DOF) system as Fig. 1. The dynamic equation is written as Eq. (1).

$$\begin{bmatrix} m_t & 0 \\ 0 & m_t \end{bmatrix}\begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \end{bmatrix} + \begin{bmatrix} 2m_t\xi\omega_n & 0 \\ 0 & 2m_t\xi\omega_n \end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} + \begin{bmatrix} m_t{\omega_n}^2 & 0 \\ 0 & m_t{\omega_n}^2 \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} a_p h_{xx}(t) & a_p h_{xy}(t) \\ a_p h_{yx}(t) & a_p h_{yy}(t) \end{bmatrix}\begin{bmatrix} -x(t)+x(t-T) \\ -y(t)+y(t-T) \end{bmatrix} \tag{1}$$

where

$$\begin{cases} h_{xx} = \sum_{j=1}^{N_f} g(\phi_j(t))\sin(\phi_j(t))[K_t\cos(\phi_j(t)) + K_n\sin(\phi_j(t))] \\ h_{xy} = \sum_{j=1}^{N_f} g(\phi_j(t))\cos(\phi_j(t))[K_t\cos(\phi_j(t)) + K_n\sin(\phi_j(t))] \\ h_{yx} = \sum_{j=1}^{N_f} -g(\phi_j(t))\sin(\phi_j(t))[K_t\sin(\phi_j(t))\text{-}K_n\cos(\phi_j(t))] \\ h_{yy} = \sum_{j=1}^{N_f} -g(\phi_j(t))\cos(\phi_j(t))[K_t\sin(\phi_j(t))\text{-}K_n\cos(\phi_j(t))] \end{cases} \tag{2}$$

where m_t, ξ, ω_n are modal mass, damping coefficient, and the natural frequency; a_p is the axial cutting depth; $x(t\text{-}T) - x(t)$ and $y(t\text{-}T) - y(t)$ represent the dynamic vibration vectors in the x and y directions at the instants $(t-T)$ and t. T is time delay period, equivalent to the tooth passing period, namely $T = 60/(N_f n)$. N_f is the number of teeth and n is the spindle speed (r/min). K_t, K_n is the tangential cutting force coefficient and radial cutting force coefficient, respectively. $g(\phi_j(t))$ is angular position of jth tooth as:

$$\phi_j(t) = (2\pi n/60)t + (j-1)\cdot 2\pi/N_f$$

The $g(\phi_j(t))$ is a screen function to judge whether jth tooth is in cutting or not as:

$$g(\phi_j) = \begin{cases} 1 & \phi_{st} < \phi_j < \phi_{ex} \\ 0 & \text{otherwise} \end{cases}$$

where ϕ_{st} and ϕ_{ex} are the start and exit angles, respectively, defined as:

$$\begin{cases} \phi_{st} = 0,\ \phi_{ex} = \arccos(1\text{-}2a_e/D) & \text{up-milling} \\ \phi_{st} = \arccos(2a_e/D\text{-}1),\ \phi_{ex} = \pi & \text{down-milling} \end{cases}$$

where a_e is the radial cutting depth and D is the diameter of the cutter.

2.2. Determination for the stability using precise integration method

Equation (1) is easily converted into a Hamiltonian system as:

$$\dot{v}(t) = A_0 v(t) + A(t)v(t) - A(t)v(t\text{-}T) \tag{3}$$

where

$$A_0 = \begin{bmatrix} -\xi\omega_n & 0 & 1/m_t & 0 \\ 0 & -\xi\omega_n & 0 & 1/m_t \\ m_t(\xi^2-1){\omega_n}^2 & 0 & -\xi\omega_n & 0 \\ 0 & m_t(\xi^2-1){\omega_n}^2 & 0 & -\xi\omega_n \end{bmatrix} \quad A(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ a_p h_{xx}(t) & a_p h_{xy}(t) & 0 & 0 \\ a_p h_{yx}(t) & a_p h_{yy}(t) & 0 & 0 \end{bmatrix}$$

$$v(t) = [x(t),\ y(t),\ m_{tx}\dot{x}(t) + m_{t\,x}\xi_x\omega_{nx}x(t),\ m_{ty}\dot{y}(t) + m_{t\,y}\xi_y\omega_{ny}y(t)]^T$$

By denoting $A(t)v(t) - A(t)v(t-T)$ byg(t), the general solution for Eq. (3) can be obtained using the ordinary differential theory.

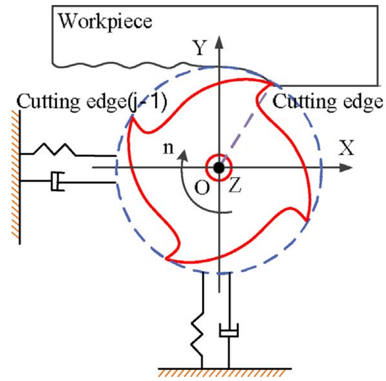

Fig. 1. A The simplified 2-DOF milling model. B Flow chart of obtaining SLD for milling process using PIM.

$$v(t) = e^{A_0\cdot(t-t_p)}\left\{v(t_p) + \int_{t_p}^{t} e^{[-A_0\cdot(\delta-t_p)]}g(\delta)d\delta\right\} \tag{4}$$

The time delay period T is equally divided into m parts, namely $T = m\cdot\tau$. In the interval of $[t_p, t_{p+1}]$, $g(t)$ can be represented as:

$$g(t) = r_0 + r_1(t - t_p) \tag{5}$$

where

$$r_0 = g(t_p) \quad r_1 = \tfrac{1}{\tau}(g(t_{p+1}) - g(t_p))$$

r_0, r_1 can be further expressed as:

$$r_0 = g(t_p) = A_p v_p - A_p v_{p-m} \quad r_1 = \tfrac{1}{\tau}(A_{p+1}v_{p+1} - A_{p+1}v_{p+1-m} - A_p v_p + A_p v_{p-m}) \tag{6}$$

Based on Eq. (4) and (5), $v(t_{p+1})$ can be gained:

$$v(t_{p+1}) = T_1[v(t_p) + {A_0}^{-1}(r_0 + {A_0}^{-1}r_1)] - {A_0}^{-1}(r_0 + {A_0}^{-1}r_1 + \tau r_1) \tag{7}$$

where $T_1 = e^{A_0}\tau$. τ is equally divided into $\Lambda = 2^{20}$ parts.

$$T_1 = e^{A_0\tau} = (e^{A_0\frac{\tau}{\Lambda}})^{\Lambda} = \left(e^{A_0\Delta t}\right)^{\Lambda} \tag{8}$$

Since $\Delta t = \frac{\tau}{\Lambda}$ is small enough, $e^{A_0}\Delta t$ is approximated as Eq. (9) by using Taylor expansion.

$$e^{A_0\Delta t} \approx I + A_0\Delta t + (A_0\Delta t)^2/2! + (A_0\Delta t)^3/3! + (A_0\Delta t)^4/4! = I + T_a \tag{9}$$

Combining with Eq. (8) and (9), the following can be carried out:

$$T_1 = (I + T_a)^{2^N} = [(I + T_a)^2]^{2^{N-1}}$$

Thereby, the non-unit matrix part of T_1 can be got by multiplying these matrix N times:

$$For\,(i = 1;\ i \le N;\ i++)\ \ T_a = 2T_a + T_a \times T_a;\ T_1 = I + T_a; \tag{10}$$

Bring Eq. (6) into Eq. (7) to get Eq. (11):

$$(I - f_1A_{p+1})v_{p+1} = (T_1 + f_2A_p)v_p - f_1A_{p+1}\cdot v_{p-m+1} - f_2A_pv_{p-m} \tag{11}$$

where f_1, f_2 are represented as:

$$f_1 = \tfrac{1}{\tau}T_a{A_0}^{-2} - {A_0}^{-1},\quad f_2 = T_a{A_0}^{-1} - f_1$$

If $(I - f_1A_{p+1})$ is invertible, Eq. (11) can be written:

$$v_{p+1} = F_0v_p - F_1v_{p-m+1} - F_2v_{p-m} \tag{12}$$

where F_0, F_1, F_2 are specified as:

$$F_0 = (I - f_1A_{p+1})^{-1}T_1 \quad F_1 = (I - f_1A_{p+1})^{-1}f_1A_{p+1} \quad F_2 = (I - f_1A_{p+1})^{-1}f_2A_p + F_2,$$