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Parameter-based spiral tool path generation for free-form surface machining

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A B S T R A C T

In computer-aided manufacturing systems, generation strategies are classified into two main types according to the generation domain of the tool path: real space and the parametric domain. Although generation methods in the parametric domain defined using a parametric surface, such as a non-uniform rational B-spline surface, have the advantage of rapid calculation, tool path variations are limited because few methods using this type of tool path generation recognize features of the surface in the parametric domain. This study proposes a new algorithm for tool path generation that can generate machining points in the parametric domain without losing information about the distances among the corresponding points in real space. Using this proposed method, a tool path with a constant pitch in real space and an efficient tool path, such as a spiral path, can be rapidly calculated. This paper explains the theory behind this method and describes its implementation using free-form surfaces composed of a single patch. The proposed method was then expanded to surfaces composed of multiple patches. In this case, because correspondence between each surface patch is required, parametric boundary representation using a half-edge structure is also proposed in this paper. Parametric boundary representation can be used to represent connections among the outer and inner trimming boundaries of each surface patch on the parametric domain. Finally, the proposed method was implemented for two surfaces composed of multiple patches, and its effectiveness was confirmed by simulating and machining these surfaces.

1. Introduction

Computer-aided manufacturing (CAM) systems are essential for the generation of tool paths on free-form surfaces. Recently, various methods of tool path generation have been studied and implemented in commercial CAM software [1]. Tool path generation strategies are classified into two main types based on their generation domain, which can be real space or a parametric domain. Methods of generating tool paths in real space are further divided into two formats based on surface representation: polygon meshes [2–8] and algebraic surfaces [9–22] such as a non-uniform rational B-spline (NURBS) surface [23]. Polygon mesh representations have come to be widely used because of the benefits of graphics processing units (GPUs) with high-speed parallel computational capabilities [24,25]. However, the fact that the number of mesh elements and the calculation cost are in a trade-off relationship is a universal problem; furthermore, regardless of how fine the polygon mesh is, because a machine tool cannot faithfully recreate folds between polygon meshes, the continuity of the free-form surface cannot be ensured.

Methods of generating tool paths in a parametric domain and using an algebraic surface representation require a high overall calculation cost; however, it is possible to adopt several tool path generation strategies that consider the features and continuity of the surface. Several superior strategies of tool path generation, including isoplanar [12,13], isoscallop [10,11,14,15,19], isoparametric [16,17], and spiral path [18–22] methods, have been proposed and applied to parametric surfaces. These methods have been researched since the 1990s, and fundamental geometrical theorems have been constructed. Recently, many studies on spiral path generation have been conducted with the integrity of the finished surface as a primary focus. For example, Hauth and Linsen [21] have proposed a generation algorithm for a double-spiral tool path using c-space. Additionally, Romero-Carrillo et al. [22] have devised a method of overcoming discontinuities in the tool path by embedding an Archimedean spiral into a linear morphing definition of the pocket in pocket milling. Their algorithm plays an important role in the algorithm developed in the present study.

Isoplanar and isoscallop strategies consider surface features, which means the number of degrees of freedom of the tool path generation is large. However, because machining points are generated in real space, the surface-surface intersection problem must be considered at all times [26,27]. Furthermore, problems involving differential equations, such as the treatment of extrema and the selection of initial points, must also be examined. In contrast, tool path generation in the parametric domain using an isoparametric path has the advantage of allowing the

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suppression of the calculation cost. Because a parametric surface $\mathbf{S}(u, v)$ is the mapping function from a point on the two-dimensional (2D) parametric domain (u, v) to a point in real space, a tool path drawn in the parametric domain can be rapidly translated to the corresponding tool path in real space. However, contrary to the case in real space, information regarding surface features cannot be easily included in the parametric domain. Therefore, special algorithms are required to consider the surface features. Suresh and Yang [10] proposed a differential equation to calculate the cutter contact (CC) path using an isoscallop method. In this method, given a point on the surface and a tool direction vector at that point, a point that satisfies a constant step in the direction perpendicular to the tool direction vector can be rapidly obtained. This means that the proposed method generates a CC path that transfers information about two directional vectors for the tool from real space to the parametric domain. Based on the method proposed by Suresh and Yang, a method of deriving differential equations based on distance information in real space to a parametric surface was established, and this enabled the differential geometric approach that generates tool paths rapidly in the parametric domain (called the parameter-based method, hereafter). However, there have been few studies on parameter-based methods since Suresh et al.'s approach was published, although there were a few studies that described improvements to their approach. The primary reason for this is that tool path generation in real space that can make use of all geometric information is becoming easy due to improvements in computational capabilities of computers, so inconvenient tool path generation in the parametric domain is becoming less widely used. However, there is still significant interest in the advantages of the parameter-based method. Due to the synergistic effect of its speed and the recent high computational performance of computers, the parameter-based method has potential for growth.

Based on the above, the present study proposes a new method for tool path generation using the parameter-based method [28,29]. In this method, the CC path characterized as a parametric curve $\mathbf{C}(t)$ is described in the parametric domain and transformed rapidly into real space considering the distance information of the free-form surface using mapping functions as differential equations. In our previous report [28], this parameter-based method was initially proposed with the shape of $\mathbf{C}(t)$ limited to a line or a circle. In a later report [29], $\mathbf{C}(t)$ was expanded to a general curve, and application to a spiral tool path was demonstrated. However, the previous reports only dealt with a single surface patch. Therefore, in this report, the method is expanded to multi-surface patches.

The remainder of this paper is organized as follows. The theorem used to develop the proposed method is explained in Section 2. Then, the proposed method applied to a spiral path that has high machining efficiency for $\mathbf{C}(t)$ is explained. Moreover, because the curve described in our previous report [29] is applicable to only a single patch, the proposed method is expanded to a surface composed of multiple patches. Under this condition, a few key points have to be considered. For example, the topological relationships between the patches, which are defined in this study using boundary representation (B-rep), are required. However, because general B-rep is performed in real space, it is not well suited to the proposed method. Therefore, a new parametric boundary representation (PB-rep) method, which is B-rep implemented in the parametric domain, is also developed in this study. Section 3 describes the PB-rep method and its implementation. The additional calculations required to ensure consistency between adjacent patches are also discussed. The implementation of the proposed method using spiral paths for two different free-form surfaces composed of multiple patches is presented in Section 4. Finally, the conclusions of this study are given in Section 5.

2. Parameter-based tool path generation

As shown in Fig. 1, When a parametric curve $\mathbf{C}(t)$ on a parametric surface $\mathbf{S}(u, v)$ is defined, the infinitesimal interval $\mathrm{d}s$ between CC points is represented as

$$\mathrm{d}s = |\mathbf{S}_u \mathrm{d}u + \mathbf{S}_v \mathrm{d}v| \tag{1}$$

Because $\mathbf{C}$ can be represented as $\mathbf{C}(t) = \mathbf{C}(u(t), v(t))$, the infinitesimal distance elements $\mathrm{d}t$, $\mathrm{d}u$, and $\mathrm{d}v$ in the parametric domain are related as

$$\frac{\mathrm{d}v}{\mathrm{d}u} = \frac{\mathrm{d}v/\mathrm{d}t}{\mathrm{d}u/\mathrm{d}t} = \frac{C_{t.v}}{C_{t.u}} \tag{2}$$

where $\mathbf{C}_t$ is the first derivative of $\mathbf{C}$ with respect to t. $C_{t.u}$ and $C_{t.v}$ are the u- and v-directions of $\mathbf{C}_t$, respectively. Combining Eqs. (1) and (2) yields

$$\frac{\mathrm{d}u}{\mathrm{d}s} = \frac{C_{t.u}}{|C_{t.u}\mathbf{S}_u + C_{t.v}\mathbf{S}_v|} \tag{3}$$

$$\frac{\mathrm{d}v}{\mathrm{d}s} = \frac{C_{t.v}}{|C_{t.u}\mathbf{S}_u + C_{t.v}\mathbf{S}_v|} \tag{4}$$

Because $\mathbf{C}_t$ depends on $\mathrm{d}t$, an equation associating $\mathrm{d}t$ with $\mathrm{d}u$ and $\mathrm{d}v$ is required. Accordingly, application of the Pythagorean theorem to the triangle formed by $\mathrm{d}t$, $\mathrm{d}u$ and $\mathrm{d}v$ as shown in Fig. 1 yields

$$\mathrm{d}t = \frac{\sqrt{\mathrm{d}u^2 + \mathrm{d}v^2}}{|\mathbf{C}_t|} \tag{5}$$

Defining the right-hand sides of Eqs. (3) and (4) as g and h yields

$$\mathrm{d}t = \frac{\sqrt{g^2 + h^2}}{|\mathbf{C}_t|}\mathrm{d}s \tag{6}$$

Eqs. (3) and (4) are first-order differential equations that characterize the relationships between the distance in real space and the parameters u and v. Using numerical calculations, such as the Runge-Kutta method, the curve $\mathbf{C}$ defined in the parametric domain can be rapidly translated to CC points ensuring a constant pitch $\mathrm{d}s$ on $\mathbf{S}$ in real space. However, $\mathrm{d}s$ does not necessarily need to be constant. Adopting a general numerical solution of the differential equations with a variable pitch using indexing for features such as the surface curvature enables tool path generation with surface features taken into consideration. However, this paper deals only with a constant $\mathrm{d}s$ as a first step in demonstrating the effectiveness of the proposed algorithm.

The definition of $\mathbf{C}$ itself is discussed here. Although many studies have proposed strategies for determining $\mathbf{C}$, in this study, $\mathbf{C}$ was defined as a spiral function. The generalized spiral function [22] is defined as

$$\mathbf{C}(t) = (1 - t)\mathbf{C}_{\mathrm{in}}(\mathrm{m}t) + t\mathbf{C}_{\mathrm{out}}(\mathrm{m}t), \quad t \in [0, 1] \tag{7}$$

where m is the number of turns in the spiral path and $\mathbf{C}_{in}$ and $\mathbf{C}_{out}$ are the inner and outer reference curves, respectively. In the present study, because the definition domain of the parameter t is fixed from 0 to 1, mt must also be fixed from 0 to 1. Therefore, (mt mod 1) is adopted as the parameter of $\mathbf{C}_{in}$ and $\mathbf{C}_{out}$. The results of implementing the proposed algorithm to create a spiral path are shown in Figs. 2 and 3 . The free-from surface shown in these figures is 3rd-order surface and is represented by 30 control points. In these cases, $\mathbf{C}_{in}$ is a point $\mathbf{p}_0 = \{u_0, v_0\}$, and $\mathbf{C}_{out}$ is the rectangular boundary defined with a primary parametric curve in the parametric domain. Then, $\mathbf{p}_0$ must be determined using an appropriate method. In conventional tool path generation methods, the operator manually selects the point. However, in this study, $\mathbf{p}_0$ was determined using the extremum search method [30]. The extremum search curve is represented as

$$\begin{cases} \psi_{\mathrm{uu}}\mathrm{d}u + \psi_{\mathrm{uv}}\mathrm{d}v = 0 \\ \psi_{\mathrm{uv}}\mathrm{d}u + \psi_{\mathrm{vv}}\mathrm{d}v = 0 \end{cases} \tag{8}$$

where $\mathbf{n}_f = [0, 0, 1]^T$ is a unit vector in the z-direction, $\psi(u, v)$ is the inner product between $\mathbf{n}_f$ and $\mathbf{S}(u, v)$, and the subscripts of ψ indicate partial derivatives with respect to u or v. Two curves can be obtained using Eq. (8), and their point of intersection is $\mathbf{p}_0$. However, if multiple intersection points are obtained, meaning there are multiple peaks and valleys in $\mathbf{S}$, only one of these points can be selected, regardless of