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Evaluation of roundness tolerance zone using measurements performed on manufactured parts: A probabilistic approach

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ABSTRACT

In the verification of the compliance with dimensional and geometrical specifications of manufactured parts, the features are frequently sampled using Coordinate Measuring Machines. The evaluation of roundness tolerance zone by using measurements performed on cylindrical work pieces is specifically dealt with. Once a finite set of points is sampled, a fitting method is adopted to estimate the parameters of the reference circle. In particular, three different methods are considered, i.e. the usual Least Squares method, the so called Probabilistic Method, and the standardized Minimum Zone method. In order to have complete information on the reference features, knowledge of their uncertainty is required. Two uncertainty evaluation techniques are examined, i.e. a theoretical approach and the bootstrap method. Then, a new procedure based on the calculation of the probability density function of the width of roundness tolerance zone is developed. The practical application to the actual measurement of circular features on cylindrical holes allows to understand the differences between the results obtained with the presented approaches. Finally, the advantages of the probabilistic approach are highlighted.

1. Introduction

In the verification of the compliance with dimensional and geometrical specifications of manufactured parts, the features are commonly sampled using specific instruments such as Coordinate Measuring Machines (CMMs). These devices give the spatial coordinates of points belonging to the surfaces of the measured object. The inspection with CMMs is a widely used method in industrial practice, due to its versatility in the verification of dimensional requirements.

This paper focuses on the problem of the evaluation of roundness tolerance [1] on cylindrical features. The current standards do not provide any clear guidelines for the roundness verification when using a CMM. Consequently, the final user has to take several decisions concerning the verification of workpiece roundness. For example, the CMM user usually decides, based on industrial practice, the sampling strategy and the methods for the estimation of the "reference circle" [2].

In this context, the Technical Committee ISO/TC 213, which has the aim of the standardization in the field of Geometrical Product Specification and Verification (GPS) project [3], focuses on the improvement of the specification and verification phases and tries to match them by postulating the "duality principle". This principle establishes that the sets of operations used in the specification phase to address variability limits are in a bi-univocal relationship with the same sets of operations used in the verification phase, in order to identify the feature that is subject to the specification and to evaluate its conformity [4]. According to the GPS project, the first verification step is the "partition", which is used to identify bounded features. Then, with the "extraction operation", a finite set of points is sampled by CMM ("extracted feature"). Finally, with an appropriate "association operation", i.e. a fitting method, an ideal feature ("associated feature") is calculated from the extracted feature [5].

Many techniques are available for the estimation of the parameters of the reference circle ("association methods"). Current standards do not establish a specific association method to be adopted.

The Least Squares method [6] is the most commonly adopted in commercial software programs, including CMM software. The Least Squares method offers the advantage of computational simplicity, but it gives solutions that may be affected by surface irregularities. Solutions more refined may instead be obtained by means of a probabilistic approach developed in the authors' Department [7]. Both methods are statistical and need to be compared with the standardized Minimum Zone method [1,8]. In fact, the values of the parameters of the features obtained from all of these methods may show significant, or even highly significant, differences. Furthermore, in order to have complete information on the reference features, knowledge of their uncertainty [9]

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is required. However, there is not any standardized procedure for uncertainty evaluation, which is univocally adopted.

Therefore, the authors' research was aimed to examine existing algorithms for uncertainty evaluation [10,11] and to develop new procedures. In particular, two uncertainty evaluation techniques were examined: the "gradient method", i.e. a theoretical approach, and the "bootstrap method" [12], i.e. a Monte Carlo technique. Then, a new procedure based on the calculation of the probability density function of the width of roundness tolerance zone [1,13] was developed. In order to understand the differences between the results obtained with the above mentioned approaches, a deep analysis was necessary. To this aim, a practical application to an actual measurement of circular features on cylindrical holes is presented. It was examined a plate with holes specifically made in the workshop of the authors' Department for the purposes of the present research. All the measurements were performed with the same CMM and the same sampling strategy. In order to meet the expected functional requirements, the width of roundness tolerance zone and its uncertainty were calculated. The differences between the methods were analyzed by comparing the corresponding results.

2. Estimation of the parameters of geometrical features

As mentioned in Section 1, current standards do not establish a specific association method for the estimation of the parameters of geometrical features. Thus, different measurement techniques may be used to estimate the form error of geometrical features [14]. In general, these techniques belong to either statistical or extreme-fit approaches [15]. Statistical methods employ all the measurement points for the estimation, while extreme-fit method estimates are based on only the extreme points. Sections 2.1 and 2.2 describe two statistical methodologies, i.e. the commonly adopted Least Squares method [6] and a refined probabilistic approach [7]. Section 2.3 instead describes the standardized Minimum Zone method [1,8], which is an extreme-fit methodology.

2.1. Least squares method

The Least Squares (LS) [6] is the most popular fit used in computational coordinate metrology [16]. LS method is based on the minimization of the sum of the squares of the error-of-fit in a given set of measured points. Let us suppose that the relation of q parameters $\theta = [\theta_1, ..., \theta_q]^T$ to the n measurements $\boldsymbol{X} = [\boldsymbol{x}_1, ..., \boldsymbol{x}_n]^T$ is given as

$$\boldsymbol{X} = \boldsymbol{F}(\theta) + \varepsilon \tag{1}$$

where $\boldsymbol{F}(\theta)$ is a nonlinear continuously differentiable observation function of θ, and ε represents errors with zero mean. The procedure has the aim of eliminating the influence of the errors ε. Given $\boldsymbol{X}$, the LS estimates the parameter θ by minimizing the sum of squares σ_0^2, which is defined as

$$\sigma_0^2 = [\boldsymbol{X} - F(\hat{\theta})]^T [\boldsymbol{X} - F(\hat{\theta})] \tag{2}$$

2.2. Probabilistic method

Let us now consider an approach that was developed in the authors' Department [7], which, from here on, will be called the Probabilistic Method (PM). This approach has the aim of extracting a nominal model from a cloud of points measured on an actual shape. The latter is interpreted as a probabilistic representation of the nominal surface, which is not necessarily described by a finite set of parameters. The definition of the parameters of a particular geometric feature is made according to the classification of invariant subsets [17] adopted in the GPS framework. The idea relies on the fact that any geometrical object in $\boldsymbol{R}^3(\boldsymbol{R}^2)$ belongs to one of the seven (three) symmetry classes, and thus the domain of all the possible geometrical features is reduced drastically. This classification is based on the invariance properties of the object, with respect to the associated rigid motion. Furthermore, the PM employs the Parzen method [18] to obtain a non-parametric estimate of the unknown probability density function (PDF) of the measurements from a finite number of samples.

The construction of the PDF originally refers to the indicator function. Thus, given a set $S \subset \boldsymbol{R}^3$ that describes the shape, the indicator function of S can be defined as $i_S: \boldsymbol{R}^3 \to \{0, 1\}, \forall P \in \boldsymbol{R}^3, i_S(P) = 1$ iff $P \in S$. A random variable, with a defined PDF p_S, describes the product boundaries

$$p_S(P) = \frac{i_S(P)}{\int i_S(Q) d^3Q} \tag{3}$$

which, by construction, is uniform on its support S. When the nominal points are unknown, and measurement points are introduced, it is possible to define the related PDF $\hat{p}_S$ (under a weak condition imposed on the noise), which reflects the incidental invariance of S and vice versa [19].

The classification allows a model dependent PDF to be defined. For each class C_i, $i = 1, ..., 7$, a model M_i and a set of reference parameters θ_i are assigned in order to produce a PDF that is invariant under the action of the group of symmetry G_i. Furthermore, the invariance is enforced by replacing the set of measured points $D = \{(x_1, y_1, z_1), (x_2, y_2, z_2), ..., (x_n, y_n, z_n)\}$ with its projection on the quotient set $\boldsymbol{R}^3/\mathrm{E_i}$, where $\mathrm{E_i}$ is the set of equivalent points with respect to the rigid motion $r \in G_i$. This projection is performed by means of a suitable parametric function $V_i(\cdot;\theta_i)$.

Then, the projected PDF is approximated by means of a consistent estimate $\tilde{p}$, which is provided by the Parzen method. In general form the PDF can be written as

$$\tilde{p}(x, y, z | M_i, \theta_i, V_i(D; \theta_i)) \tag{4}$$

Parameters of a model M_i are estimated by solving the problem for the model likelihood

$$\tilde{\theta}_i = \underset{\theta_i}{\mathrm{argmax}} L(\theta_i ; D) \tag{5}$$

where

$$L(\theta_i ; D) = \sum_{j=1}^{n} \log[\tilde{p}(x_j, y_j, z_j | \theta_i, V_i(D ; \theta_i))] \tag{6}$$

2.3. Minimum zone method

The Minimum Zone (MZ) method is an extreme-fit procedure which arises naturally from the definition of the tolerance zone, according to both ISO 1101:2012 [1] and ASME Y 14.5:2009 [8].

Standards suggest that the MZ criterion should be applied, where possible, to evaluate form errors. The drawback of techniques based on the MZ method is that they require the solution of a non-linear problem. The complexity, and consequently the computation time of MZ algorithms are very sensitive to the number of sample points [20].

The algorithms developed for MZ error evaluation can be classified in two different families, according to the nature of the method used to solve the MZ problem. On one hand, there are numerical methods [20,21], e.g. an approach based on non-linear optimization was followed in this work. On the other hand, there are computational-geometry-based techniques that rely on the computation of a convex hull for a given finite set of measurement points [22].

In the case of the roundness evaluation that is considered in this work, the MZ procedure can be described as follows. Given a set $D = \{\boldsymbol{z}_1, \boldsymbol{z}_2, ..., \boldsymbol{z}_n\}$ of n measurement points $\boldsymbol{z}_i = (x_i, y_i)$, $i = 1, ..., n$ that are extracted from a circular profile, the width of roundness tolerance zone (radial extreme difference), which is parameter (the center