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# Modeling and analysis of sub-aperture tool influence functions for polishing curved surfaces

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ABSTRACT

Computer controlled optical surfacing requires accurate tool influence functions (TIFs) for deterministic fabrication of smooth curved surfaces. In the present paper, the three dimensional TIFs are investigated separately from their radial profiles, which can be simplified into circular arcs. TIFs are found significantly affected by the form deviation between the polishing pad and the workpiece, regardless of the specific shape of the workpiece. A new analytical TIF model is proposed as a function associated with the relative curvature difference. The condition to guarantee single-peak TIFs is then derived, which is of essence for stable planning of polishing paths. Thereby appropriate processing parameters can be selected accordingly. Practical experimental results demonstrate that the new model can reliably predict the TIFs in different situations, and the modelling error is greatly improved compared to the conventional methods.

## 1. Introduction

Complex curved surfaces such as aspheric and freeform surfaces with high form accuracy have become the key features of optical components increasingly employed in the fields of astronomical observation and opto-electronic industries [1–3]. Polishing is an essential step of optical fabrication. The form errors with respect to the nominal shapes are reduced with the computer controlled optical surfacing (CCOS) [4,5] first proposed by Itek Inc. in 1970s. In CCOS the polishing process is regarded as a convolutional operation between the tool influence function (TIF) and the dwell time function [6,7]. Thus TIF plays an important role in the path planning and quality control of CCOS polishing. Usually a single-peak TIF is required to obtain higher surface quality in deterministic fabrication.

At present, a commonly used method to determine polishing TIFs is the trial-and-error approach [8,9]. The polishing tool is fixed first at a particular location to polish the workpiece for some time, and then the TIF can be calculated based on the relative difference between the surface forms before and after fabrication. For the small-tool polishing, TIFs are significantly affected by the degree of conformance between the forms of the polishing pad and the workpiece [10]. As a result this method is no longer suitable for polishing complex curved surfaces because the local forms vary at different regions of the workpiece surface, which leads to a region-dependent TIF. As it is infeasible to measure the TIF at every position of the workpiece, hence in practice only the TIFs sampled at several points are sampled for planning the polishing path. In this approach the form errors of the workpieces cannot converge rapidly due to the inaccurate estimation of actual TIFs. Therefore an accurate model is urgently required to predict the TIFs in the polishing of complex curved surfaces.

This paper is organized as follows. The theoretical background of the sub-aperture polishing is presented in Section 2. The proposed TIF model is derived in Section 3 and the allowable range of the curvature difference is given for single-peak TIFs in Section 4. Section 5 presents experimental demonstration. Finally the paper is summarized in Section 6.

## 2. Theoretical background

In the field of optical polishing, the material removal depth per unit time can be calculated by the Preston equation [11]

$$\mathrm{d}z(x, y) = k \cdot p(x, y) \cdot V(x, y) \cdot \mathrm{d}t \tag{1}$$

where $\mathrm{d}z(x,y)$ is the material removal depth, and $k$ is the Preston coefficient related to the particular polishing conditions. $p(x,y)$ denotes the pressure in the contact region and $V(x,y)$ is the relative velocity between the workpiece and the pad.

The relative velocity can be obtained by the characteristics of the dual-rotation motion of the polishing tool [12]. The kinematic relation is shown in Fig. 1. The center of the orbital motion is $O_1$ and the center

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**Nomenclature**

| | |
|---|---|
| $z$ | Material removal amount |
| $k$ | the preston coefficient |
| $p$ | the pressure distribution |
| $V$ | the relative velocity distribution of the workpiece |
| $f$ | the speed ratio of polishing tool |
| $e$ | the eccentricity of polishing tool |
| $\omega_1$ | the angular velocity of the orbital motion |
| $\omega_2$ | the angular velocity of the spin motion |
| $r_0$ | the radius of the tool |
| ρ | Eccentric distance |
| $\delta$ | the form deviation between the pad and the workpiece |
| $F$ | the magnitude of the force applied on the polishing pad |
| $E$ | the young's modulus of the pad |
| $L$ | the thickness of the pad |
| $d_0$ | the displacement of the polishing head |
| $C$ | the maximal principal curvatures of the form deviations |
| $C'$ | the minimal principal curvatures of the form deviations |
| $r_p$ | the radius of curvature of the polishing pad |
| $r_w$ | the radius of curvature of the workpiece |
| $\kappa_p$ | the curvature of the polishing pad |
| $\kappa_w$ | the curvature of the workpiece |
| $\Delta\kappa$ | the curvature difference between the polishing pad and the workpiece |
| $\sigma$ | a measure to determine whether the TIF is of single-peak |
| $\Delta$ | the relative error of the TIF model |

of the tool is $O_2$. The angular velocities of the orbital motion and the spin motion are $\omega_1$ and $\omega_2$, and $r_0$ and ρ denote the tool radius and the eccentric distance, respectively.

According to Fig. 1, the relative velocity distribution can be derived by the following equation,

$$V(R) = \frac{\omega_1}{2\pi}\int_{-\theta_0}^{\theta_0}[R^2(1+f)^2 + r_0^2 f^2 e^2 - 2Rr_0 fe(1+f)\cos\ \theta]^{\frac{1}{2}}\,d\theta$$
$$\text{with} f = \frac{\omega_2}{\omega_1},\ e = \frac{\rho}{r_0},\ \theta_0 = \arccos\left(\frac{R^2 + (e^2-1)r_0^2}{2eRr_0}\right)$$
$$\text{and} R \in [0,\ (1+e)r_0] \tag{2}$$

Here $R$ is radius coordinate associated with an arbitrary point $P$ in the polar coordinate system originated at $O_1$. $f$ denotes the speed ratio and $e$ is the eccentricity.

## 3. Modeling of TIFs for different form deviations

### 3.1. TIF model for curved surface

The TIFs for polishing complex curved surfaces are significantly affected by the degree of conformance between the forms of the polishing pad and the workpiece, which leads to the region-dependent pressure distribution, as shown in Fig. 2.

Considering the form deviations between the pad and the workpiece are usually small in practice, thus the shear stress can be neglected in calculation, and only the normal stress is considered. According to the material mechanics theory [13], it is intuitive to understand that the stress is proportional to the relative form deviations; hereafter the pressure distribution can be calculated by the following equations

$$\begin{cases} p(x,y) = u(d_0 - \delta(x,y))\cdot\frac{E}{L} \\ F = \iint\limits_S u(d_0 - \delta(x,y))\cdot\frac{E}{L}\mathrm{d}S \end{cases} \quad \text{with u(x)} = \begin{cases} x & x \ge 0 \\ 0 & x < 0 \end{cases} \tag{3}$$

where $\delta(x,y)$ denotes the form deviation between the pad and the workpiece and $F$ is the magnitude of the force applied on the polishing pad. $E$ and $L$ are the Young's modulus and thickness of the pad. $d_0$ is the displacement of the pad when the force is applied.

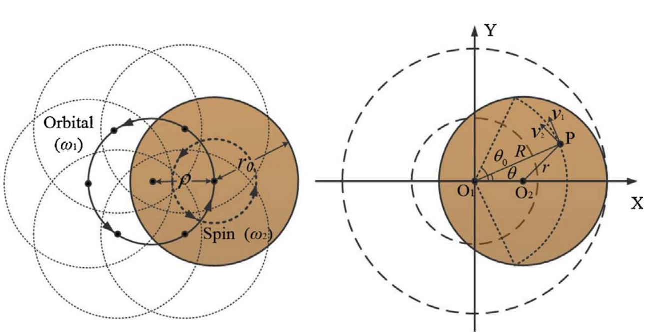


**Fig. 1.** Schematic diagram of the dual-rotation motion.

In practice the polishing pad can tilt according to the local slope at the contacting area of the workpiece, thus the form deviations are always measured along the normal vectors of the workpiece. The maximal and minimal principal curvatures $C$ and $C'$ of the form deviations are calculated, and then the form deviations can be classified into three cases accordingly [14], as shown in Fig. 3.

Based on the pressure model and Equations (1)–(3), the TIFs of complex curved surfaces can be calculated. The normalized TIFs corresponding to the above three classes of form deviations are shown in Fig. 4, with eccentricity $e = 0.6$, speed ratio $f = -3$, force $F = 10$ N, pad thickness $L = 2$ mm, Young's modulus $E = 80$ MPa and tool radius $r_0 = 12.5$ mm.

### 3.2. Simplification of the TIF model

In most cases, the surfaces of polished workpieces are slow-varying smooth surfaces. Thus the radial profiles of the pad and the corresponding local area at the workpiece can be regarded as circular arcs, then a radial profile of the TIF of complex curved surfaces is equivalent to that of the TIF of spherical surfaces with the same radius of curvature. This means the above TIF model can be simplified by investigating its two-dimensional profile obtained by polishing simple spherical surfaces. Henceforth the pressure distribution $p(x,y)$ and form deviation $\delta(x,y)$ are rewritten as $p(r)$ and $\delta(r)$. Assume that the radii of curvature of the polishing pad and the workpiece are $r_p$ and $r_w$, respectively. The schematic diagram is shown in Fig. 5.

The form deviation can be expressed as Eq. (4)

$$\delta(r) = \mathrm{sgn}(r_p)\cdot\left(|r_p| - \sqrt{r_p^2 - r^2}\right) - \mathrm{sgn}(r_w)\cdot(|r_w| - \sqrt{r_w^2 - r^2})$$
$$\text{with}\quad \mathrm{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \tag{4}$$

where $r$ is the radius coordinate associated with an arbitrary point in the polar coordinate system originated at $O_2$. Here the function sgn($x$) reflects the concavity of surface shapes.

For most optical components, the radius of curvature is generally much greater than the aperture of the component, i.e. $|r_a| \gg r$. Thus the circular arc can be approximated into a quadratic function

$$\mathrm{sgn}(r_a)\cdot(|r_a| - \sqrt{r_a^2 - r^2}) \approx \frac{1}{2r_a}r^2 = \frac{1}{2}\kappa_a r^2 \tag{5}$$

where $\kappa$ represents the curvature, which is the reciprocal of the radius of curvature.

Substituting Eq. (5) into Eq. (4), we can obtain